A phenomenological model for turbulent heat flux in high-speed flows with shock-induced flow separation

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High-speed flows with shock waves impinging on turbulent boundary layers pose severe challenge to current computational methods and models. Specifically, the peak wall heat flux is grossly over-predicted by Reynolds-averaged Navier Stokes simulations using conventional turbulence models. This is because of the constant Prandtl number assumption, which fails in the presence of strong adverse pressure gradient of the shock waves. Experimental data suggest a reduction of the turbulent Prandtl number in boundary layers subjected to adverse pressure gradient. We use a phenomenological approach to develop an algebraic model based on the available data, and cast it in a form that can be used in high-speed flows with shock-induced flow separation. The shock-unsteadiness k-ω model is used as the baseline, since it gives good prediction of flow separation and the regions of adverse pressure gradient. The new model gives marked improvement in the peak heat flux prediction near the reattachment point. The formulation is applicable to both attached and separated flows. Additionally, the simplicity of the formulation makes it easily implementable in existing CFD codes.

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Nomenclature

Pr_T = Turbulent Prandtl number

k = Turbulent kinetic energy

ω = Specific dissipation rate of turbulent kinetic energy

q_w = Wall heat flux

β = Deflection angle

ν = Kinematic viscosity

y^+ = Dimensionless distance from the wall

ν_T = Turbulent eddy viscosity

Pr = Prandtl number

Re_θ = Momentum thickness Reynolds number

H = Velocity shape factor

δ = Boundary-layer thickness

α_T = Eddy diffusivity of heat

μ = Dynamic viscosity

f_{APG} = Adverse pressure gradient function

S_{ij} = Symmetric part of mean strain rate tensor

Subscripts

∞ = Free-stream condition

w = Wall condition

1. Introduction

High-value engineering applications, such as gas-turbine components, external body of supersonic vehicles and rocket nozzles, require detailed computational analysis before further resources are committed to experimentation. Several of these applications include shock/boundary-layer interactions (SBLIs). When the shock waves are strong, the flow may get separated from the surface at one point and re-attach later, generating additional shock and expansion waves. Computational study of these complex flow-structures is often done using the Reynolds Averaged

Navier Stokes (RANS) simulations, owing to its low computation cost [1, 2]. Several studies have been conducted to investigate parameters like surface pressure rise, effect of wall temperature [3] in SBLIs with varying conditions and configurations [4, 5]. Among these, predictions for SBLIs with strong shock waves have been found unsatisfactory, especially for eddy viscosity based models. This is because current two-equation turbulence models are unable to predict the flow separation correctly. Reynolds stress model (RSM) are found to perform better in shock-separated flows [6]. The majority of two-equation RANS models over-predict the peak heat flux by considerable margins [7]. In this work, we aim to improve the surface heat flux prediction capabilities of turbulence model, utilized for RANS simulations. We focus on two-equation models based on eddy viscosity assumption.

Transport of heat in standard turbulence models is modelled through gradient-diffusion hypothesis, with thermal conductivity having molecular and turbulent components. The turbulent part of thermal conductivity is usually written in terms of the turbulent Prandtl number (Pr_T), which is calculated based on the Strong Reynolds Analogy. It states that the value of velocity-temperature correlation coefficient is -1, which leads to a theoretical value of $Pr_T = 1$ [8]. A value of 0.89 produces results that are in good agreement with the experimental data for turbulent boundary layers, and is conventionally used for SBLI. However, Pr_T is a turbulent quantity and depends upon local turbulence field. The ability of the constant Pr_T approach is questionable, especially in the highly non-equilibrium turbulence encountered in SBLI [9].

Among the variable Pr_T models proposed in the past, many are algebraic and are constructed through the phenomenological approach. Two models of such kind, devised by Kays [10] and Blackwell [11], have been discussed extensively in a subsequent section. These models utilize the available experiment and DNS data on Pr_T variation through various regions of the boundary layer, namely viscous sub-layer region, log region and wake region. Kays's model [10] is valid for zero pressure gradient (ZPG) boundary layers. It produces Pr_T variation in the inner region (viscous sub-layer region and log region) through an empirical equation that utilizes eddy-viscosity. Blackwell's model [11] varies Pr_T in the entire boundary layer and works in adverse pressure gradient (APG) boundary layers, in addition to ZPG boundary layers. It divides the boundary layer into inner and outer regions, and presents a separate formulation for Pr_T variation for each region.

The range of application of the algebraic models is presently limited to attached boundary layer flows with adverse pressure gradients. This limitation does not exist in the case of transport equation based variable Pr_T models, which solve two additional transport equations for temperature variance and its dissipation rate [12, 13]. The solution of extra equations thus requires more computational power and invariably increases the cost of computation. Additionally, inclusion of transport equation based models in existing CFD codes is difficult when compared to algebraic models. By comparison, algebraic models are attractive from an application point of view. This is because they add negligible computational cost and can be easily implemented in existing CFD codes.

In this work, we propose an algebraic variable Pr_T model applicable to SBLIs with flow-separation. The model is developed by extending the methodology of Kays and Blackwell to SBLI flows. Experiment data is used to propose an empirical form that is simpler than the earlier models. The proposed variable Pr_T model is built upon the shock-unsteadiness (SU) k-ω turbulence model [14]. Compared to conventional models, the shock-unsteadiness correction gives significant improvement in predicting the separation bubble size and the surface pressure distribution in SBLI flows [15]. The SU k-ω model has been extensively validated for different SBLI configurations for a range of Mach numbers [16, 17]. A brief review of the governing equations and the shock-unsteadiness modified k-ω model are presented in the next section. The numerical method and boundary conditions used in the simulations, and the SBLI test cases are also presented. Next, we propose the variable Pr_T model in the model development section, followed by computed results and their comparison with experimental data.

2. Simulation methodology

We solve the Reynolds-averaged Navier-Stokes equations for the mean flow and the transport equations for the turbulence quantities. The standard k-ω model of Wilcox [9] and the shock unsteadiness modified k-ω model of Sinha et al. [15] are utilized for turbulence closure. The turbulence models do not include any compressibility corrections in the form of dilatation and pressure dilatation models of Sarkar and Zeman, as described by Wilcox [18]. They are found to deteriorate model predictions in the undisturbed boundary layer upstream of the shock-boundary layer interaction [19, 20].

The accuracy of standard RANS turbulence models, like k-ϵ and k-ω, is limited in high-speed flows involving strong shock waves. This is due to the over-amplification of turbulent kinetic energy across the shock wave by the production term of eddy viscosity models. The standard models cannot correctly capture the physics of the interaction of vorticity modes with the shock wave. Also the eddy viscosity model cannot reproduce the resulting anisotropy correctly, which is independent of the mean shear. The over estimation of TKE leads to a more energized boundary layer, which is able to sustain the adverse pressure gradient created by the shock waves for longer, thus delaying flow separation. Sinha et al. [14] model the unsteady effect of shock waves in the otherwise steady RANS framework by rectifying the production term of the eddy viscosity models and thus eliminate the over-amplification of turbulent kinetic energy across shock waves. The shock-unsteadiness modified turbulence models are found to improve the computed flow topology significantly, thereby giving better predictions for surface pressure and separation bubble size, in flows with shock-shock and shock-boundary layer interactions [15–17]. The shock-unsteadiness modified k-ω model, described in Pasha et al. [16], is used as a baseline model for developing the variable Pr_T model.

A finite volume formulation is used to discretise the governing mean flow equations, where the inviscid fluxes are solved using a modified, low-dissipation form of the Steger-Warming flux splitting method [21]. This method reduces the numerical dissipation, and is found to be useful for high-speed flows with strong shock waves and viscous-inviscid interactions with boundary layers. The discretization method is second-order accurate in space; the details of the formulation can be found in Ref. [22]. The viscous fluxes and the turbulent source terms are calculated using second-order accurate central difference method. The implicit Data Parallel Line Relaxation (DPLR) method of Wright et al. [23] is used to integrate the equations in time to reach a steady-state solution. The method has been validated in a wide range of supersonic and hypersonic flows [15–17].

Experiments were conducted by Schulein [24] for oblique shock wave/turbulent boundary layer interaction flows with shock deflection angles of 6^o, 10^o and 14^o. All the tests were performed at Mach 5. The experimental configuration, shown in Fig. 1, consists of a flat plate and a shock generator with deflection angle β. The position of the deflector was varied for each test case to

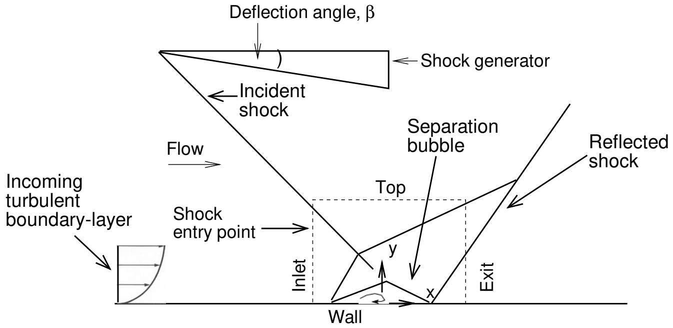

Fig. 1 Experimental configuration of Schulein [24] to study the interaction of an oblique shock with a turbulent boundary layer.

impinge the inviscid shock at a fixed location of 350 mm from the plate leading edge. The incident shock strength increases with higher deflection angle, resulting in a stronger interaction with the turbulent boundary layer developed over the flat plate. The inflow unit Reynolds number, Reynolds number based on momentum thickness Re_θ, static pressure and temperature are 37×10^6 m^{-1}, 5800, 4008.5 $\mathrm{N/m}^2$ and 68.3 K respectively. The flat plate surface temperature is 300 K except with adiabatic heating at the model inserts used for heat transfer measurements. Dry air was taken as the test medium, with perfect gas assumption. Surface properties like pressure, skin friction and heat transfer rates were obtained along the flat plate in the interaction region.

The computational domain is shown in Fig. 1 by dashed line and it extends to about 85 mm upstream and 150 mm downstream of the shock impingement point. Boundary layer thickness upstream of the shock impingement is 5.9 mm. No slip and isothermal boundary conditions are used at the wall, while extrapolation condition is applied at the exit boundary of the domain. Inlet profiles are obtained from separate flat plate simulation; the details can be found in Pasha et al. [16]. The boundary conditions for turbulence quantities are as per Menter [25]. At the wall, $k = 0$ and $\omega = 60\nu_w/\beta_1 \Delta y_1^2$, where ν_w is kinematic viscosity at the wall, $\beta_1 = 3/40$ and Δy_1 is the normal distance to the grid point nearest to the wall.

3. Model development

The variation of the turbulent Prandtl number in boundary layers have been extensively studied; see Ref. [10] for a comprehensive survey. Experimental data covering a range of Reynolds number, for different fluids are reported in literature. Limited amount of DNS data is also available for comparison. The common findings from the experimental and DNS data have been cast into empirical models for the turbulent Prandtl number. We present two prominent examples, and use their methodology to develop a variable Pr_T model for SBLI applications. We restrict to algebraic formulations, because of their low computational cost and easy implementation in CFD codes.

Kays [10] studied the behaviour of Pr_T in the three characteristic regions of zero pressure gradient boundary layers. The data suggests a constant turbulent Prandtl number of around 0.85 in the log layer ($30 < y^+ < 200$). Here, y^+ is the non-dimensional distance from the wall, computed using the wall shear stress and the kinematic viscosity of the fluid. In the viscous sub-layer ($0 < y^+ < 30$), Pr_T is higher than 0.85 and has increasing trend as we approach the wall. For the wake region ($y^+ > 200$), experiments and DNS data suggest a value of 0.5 to 0.7 for the turbulent Prandtl number. Based on the data, Kays and Crawford proposed a model for zero pressure gradient turbulent boundary layers [10]. The model is applicable in the inner region (viscous sub-layer and log layer) and is given by,

$$Pr_T = \left(0.5882 + 0.228\left(\frac{\nu_T}{\nu}\right) - 0.0441\left(\frac{\nu_T}{\nu}\right)^2\left[1 - e^{-\frac{5.165}{\left(\frac{\nu_T}{\nu}\right)}}\right]\right)^{-1} \quad (1)$$

which predicts Pr_T in the viscous sub-layer accurately, and gives a value of 0.85 in the log region.

Blackwell [11] studied turbulent boundary layers subjected to adverse pressure gradient. The data, once again, suggests a constant value of Pr_T in the log-layer. However, adverse pressure gradient causes Pr_T to be lower than its value in zero pressure gradient boundary layers. A higher adverse pressure gradient leads to a lower value of Pr_T. In the wake region, the Pr_T value is in the range 0.5 - 0.7, as earlier. The following model was proposed for the inner part of the boundary layer, covering the viscous sub-layer and the log layer.

$$Pr_T(\text{Inner}) = 0.8\left[\frac{\frac{0.5\nu}{\nu_T Pr}}{1 - e^{\frac{-0.5\nu}{\nu_T Pr}}}\right]\left[1000\left(\frac{du_\infty}{dx}\right)\left(\frac{\nu}{u_\infty^2}\right)\left(\frac{Re_\theta}{H-1}\right) + 2\right]^{0.17} \quad (2)$$

The factor $\left(\frac{du_\infty}{dx}\right)$, normalized by $\left(\frac{\nu}{u_\infty^2}\right)$, brings in the effect of adverse pressure gradient. Pr_T is obtained by scaling $\left(\frac{du_\infty}{dx}\right)$ with $\left(\frac{Re_\theta}{H-1}\right)$. The magnitude of velocity gradient $\left(\frac{du_\infty}{dx}\right)$ decreases along x, whereas $\left(\frac{Re_\theta}{H-1}\right)$ increases along the streamwise direction. Their combination produces a net effect of Pr_T being constant in the log region. The first term in the above equation containing eddy viscosity is active in the viscous sub-layer region, whereas the second part contributes to the Pr_T value in the log layer. The Pr_T formulation for the outer region is given as

$$Pr_T(\text{Outer}) = 0.5 + b_1 \left[1 - \left(\frac{y}{\delta}\right)^2\right] \tag{3}$$

where Pr_T approaches 0.5 at the boundary layer edge, while maintaining continuity between inner and outer regions using the model parameter b_1.

Extending the methodology of Kays [10] and Blackwell [11], we develop a variable Pr_T model for shock-boundary layer interactions. Regions of shock interaction are marked by severe adverse pressure gradient, and we define a function f_{APG} to identify such regions in the flow.

$$f_{APG} = \text{max.} \left\{ \tanh 4 \left(\frac{dp}{dx} \frac{\delta_0}{P_\infty} - 0.05 \right), 0 \right\} \tag{4}$$

where P_∞ is the freestream pressure and δ_0 is the undisturbed boundary layer thickness, both quantities are known a priori. The factor 0.05 eliminates small fluctuations in the pressure gradient. The APG function takes values close to 1 in the regions of shock waves, and is zero in favourable and zero pressure gradient conditions.

In the log region, Pr_T has a strong correlation with the magnitude of adverse pressure gradient. Therefore, the variable Pr_T model is defined in terms of the pressure gradient dp/dx, which is normalized by the parameters k, ω, ν and ρ. These are identified as independent variables for modeling Pr_T using the phenomenological approach. Pr_T, by definition, is a ratio of two turbulence dependent quantities – ν_T and α_T. The kinematic eddy viscosity ν_T is calculated in terms of k and ω, which are utilized in the variable Pr_T formulation. In high-speed flows, basic fluid properties, such as μ, can vary strongly in the boundary layer. Consequently μ, written in terms of ν and ρ, is included among the parameters for normalizing dp/dx. The variables k, ω, ν, ρ and dp/dx produce a non-dimensional number: $\sqrt{\frac{\nu}{\omega}} \left(\frac{1}{\rho k}\right) \frac{dp}{dx}$, which is largely constant in the log region of a boundary layer and is used to define the local Pr_T value. It has been found to be performing adequately for the cases of SBLI considered in this work.

$$Pr_T = 0.89 - f_{APG} \left(\frac{1}{3} \left[\sqrt{\frac{\nu}{\omega}} \left(\frac{1}{\rho k}\right) \frac{dp}{dx} \right]^{\frac{1}{8}} \right) \tag{5}$$

Here, f_{APG} ensures that Pr_T becomes 0.89 in the zero and favourable pressure gradient conditions, but is lower in the case of adverse pressure gradient. The value is limited to $Pr_T > 0.5$, as this is reported to be the lowest value observed in the wake region of turbulent boundary layers.

Kays [10] reports that using a variable Pr_T model in the log region tends to be sufficient for calculating heat transfer at the wall. Our studies suggest that varying Pr_T in the viscous sub-layer and outer regions tends to have a negligible effect on the surface heat flux. This is because the viscous sub-layer region is largely laminar. Therefore, turbulence does not dictate the heat flux in a significant way in this region, making the turbulence quantities like ν_T and Pr_T unimportant. The outer region is far away from the wall and the gradients in the outer region fail to have a strong effect at the wall surface. The variable Pr_T model, devised for the log region, is therefore applied to the entire boundary layer for the sake of simplicity.

We note that the current model does not include integral parameters such as Re_θ, as these are difficult to compute repeatedly in an ongoing simulation. Also, the Pr_T formulation does not depend on the local boundary layer thickness δ and y^+, which are ill-defined in separated flows encountered in strong SBLI. Older variable Pr_T models were built upon the mixing length model. By comparison, the current model uses the more robust two-equation framework. It is is based on the k-ω model, and can easily be extended to k-ϵ and other two-equation models. Further, the variable Pr_T model is built upon the shock-unsteadiness k-ω turbulence model, which gives good prediction of the separation bubble, the shock structure and the regions of adverse pressure gradient in the flow.

We also note that the model formulation is empirical in nature, and therefore may not be valid for more complex scenarios involving chemically reacting flows and flows with wall suction/blowing. Also, there is some evidence that the log-layer Pr_T value increases in the presence of favourable pressure gradient. The current model does not incorporate this trend, as favourable pressure gradient is not encountered in the regions of high heat transfer in SBLI flows.

4. Results

The new turbulent heat flux model is applied to the oblique shock impingement SBLI flows reported by Schulein [24]. Three cases (in section 2) with progressively decreasing shock strength are presented, and the effect of the variable Pr_T model on the wall heat transfer predictions is compared with experimental data and the predictions of existing turbulence models. The simulations are performed on 300×400 computational grid, based on a previous study of the same configuration by Pasha and Sinha [16]. The non-dimensional spacings along the wall normal and streamwise direction, i.e., Δy^+ and Δx^+ at the interaction region are 0.5 and 5.4, respectively. The maximum value of Δy^+ is 5.9 at the edge of the boundary-layer.

Sensitivity of the heat flux predictions to the model parameters is presented subsequently, along with a direct comparison of the present results with those of other variable Pr_T models in literature. Additional validation cases at varying Mach number and different SBLI configuration are also reported.

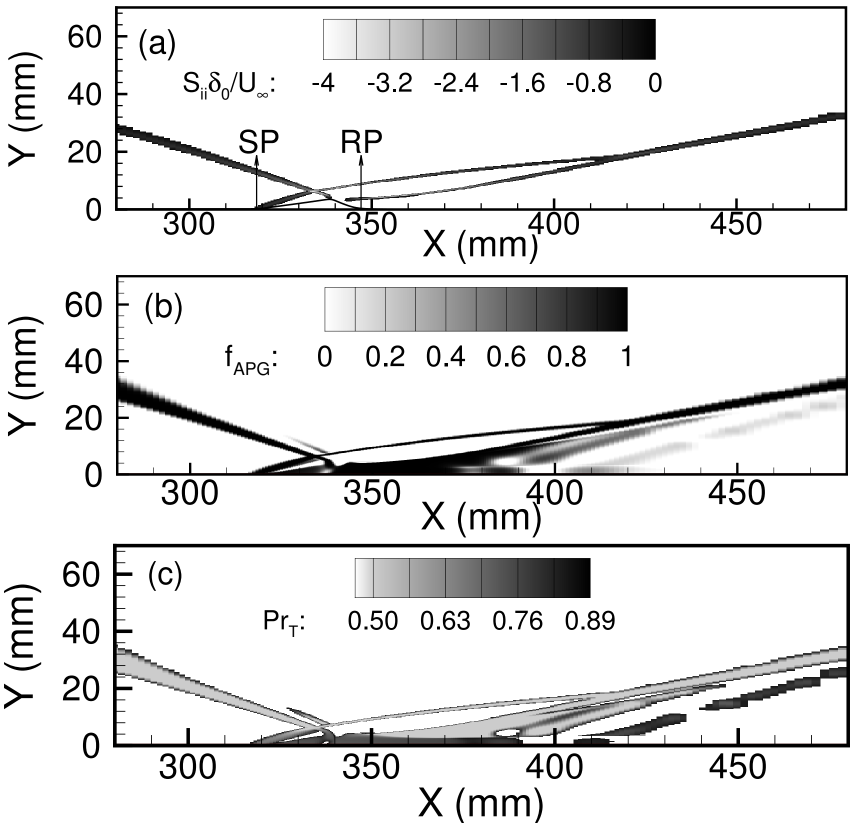

Fig. 2 Distribution of (a) normalized mean dilatation, (b) f_{APG} function and (c) turbulent Prandtl number for $\beta = 14^o$ and $M_\infty = 5$ using the SU k-ω and variable Pr_T models.

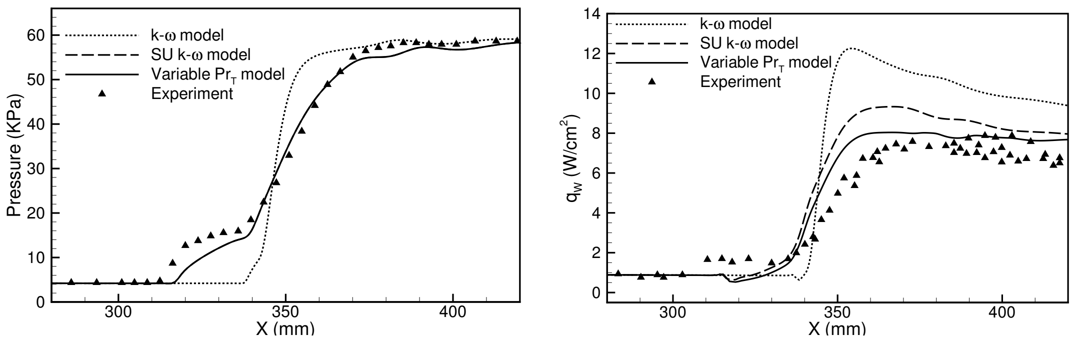

Fig. 3 Comparison of (a) surface pressure and (b) wall heat flux for $\beta = 14^o$ and $M_\infty = 5$ using standard k-ω, shock-unsteadiness k-ω and variable Pr_T models with the experimental data of Schulein [24].

14 degree case

The shock pattern computed in the 14 degree SBLI case is presented in Fig. 2(a). The shock-unsteadiness k-ω model has been utilized for the simulation. The limiting streamline is shown, and the separation and reattachment points are identified as SP and RP respectively. Further details can be found in [16]. The f_{APG} function (Fig. 2(b)) identifies the regions of adverse pressure gradient, which correspond to the shock waves and the SBLI region. Experimental data of wall pressure (in Fig. 3(a)) corroborates this plot, where pressure rises from the undisturbed boundary layer value at the separation point, and then further rises in the reattachment region (up to x = 390 mm). The simulation results obtained using the shock-unsteadiness k-ω model replicate this trend. By comparison, the standard k-ω model delays flow separation and gives a much smaller recirculation bubble. Matching the experimental pressure gradient in the reattachment region is important for the application of the variable Pr_T model, as this coincides with the location of the high heat transfer rate to the surface. We exploit the advantages of the SU k-ω model in this regard.

The distribution of Pr_T is shown in Fig. 2(c), and it highlights regions with $0.5 < Pr_T < 0.89$. The separation shock leads to an increase in adverse pressure gradient and Pr_T becomes 0.66 downstream of the shock. The Pr_T in the separation bubble is 0.74 approximately. As the flow reattaches and the boundary layer develops, the adverse pressure gradient magnitude decreases. Consequently, the Pr_T value goes from 0.74 to 0.8 in the region of reattachment, achieving a value of 0.89 eventually. The peak surface heat flux (Fig. 3(b)) obtained from the variable Pr_T model is lower than the constant Pr_T SU k-ω model prediction and is comparable to experimental values. The streamwise variation for surface heat flux in the recovering boundary layer is also well-predicted. By comparison, the standard k-ω model over-predicts the peak heat transfer by about 70%. The surface pressure obtained using SU k-ω and variable Pr_T model are identical and the dashed & solid lines are overlapping in Fig. 3(a). Also the Variable Pr_T model has no effect on c_f and the results are identical of that obtained using SU k-ω model [16].

A discrepancy between experimental and computed heat flux distribution in the separated region is observed for all models (constant Pr_T and variable Pr_T models). Experimental data shows a rise in the local heat flux value at separation point, whereas the models predict a drop due to boundary layer separation. Brown et al. [26] show that the empirical relation proposed by Back [27] can rectify the problem to some extent.

10 degree case

The variation of Pr_T in the SBLI region corresponding to the 10 degree shock generator is shown in Fig. 4. It is qualitatively similar to that in the 14 degree case. The only difference is the smaller separation bubble caused by a weaker incident shock. The region of adverse pressure gradient and the corresponding low Pr_T region is smaller compared to that in Fig. 2(c). The shock and expansion wave structure, and the distribution of f_{APG} in the 10 and 14 degree cases are qualitatively similar, and hence not repeated here. Details of the flow structure can be found in Ref. [16].

The size of separation bubble in the 10 degree SBLI case, though large enough to affect the heat flux, is small when compared to the previous case. Characteristics exhibited by each of the models, namely k-ω model, SU k-ω model and the variable Pr_T model, for the 14 degree SBLI are also present in the 10 degree case. In particular, standard k-ω model under-predicts flow separation and over-predicts surface heat flux when compared to the experiments. An improvement in the separation bubble length and surface heat flux is obtained through the SU k-ω model. The variable Pr_T model reduces the Pr_T in the adverse pressure gradient region to produce a better surface heat flux prediction in the reattachment region, without affecting surface pressure; see Fig. 5.

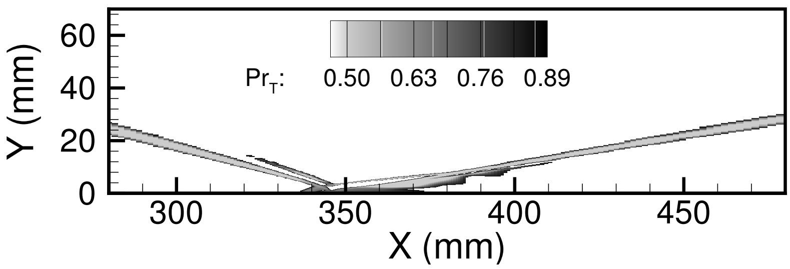

Fig. 4 Distribution of the turbulent Prandtl number computed for the 10 degree SBLI case using the variable Pr_T model.

Schulein [24] reports a wall temperature of 300 ± 5 K for the SBLI cases, with additional adiabatic heating at the model inserts used for heat transfer measurement. This can raise the surface temperature locally. Fig. 5(b) shows the effect of wall temperature on heat transfer, where T_{wall} is increased by 5, 10 and 15 K. A higher wall temperature reduces the surface heat transfer rate and it brings the predictions closer to the experimental data. Similar trends are also observed in the 14 and 6 deg. interactions.

6 degree case

The important feature of the 6 degree SBLI case is that the region of adverse pressure gradient and the corresponding low Pr_T region (see Fig. 6) are small. This is because of the marginal flow

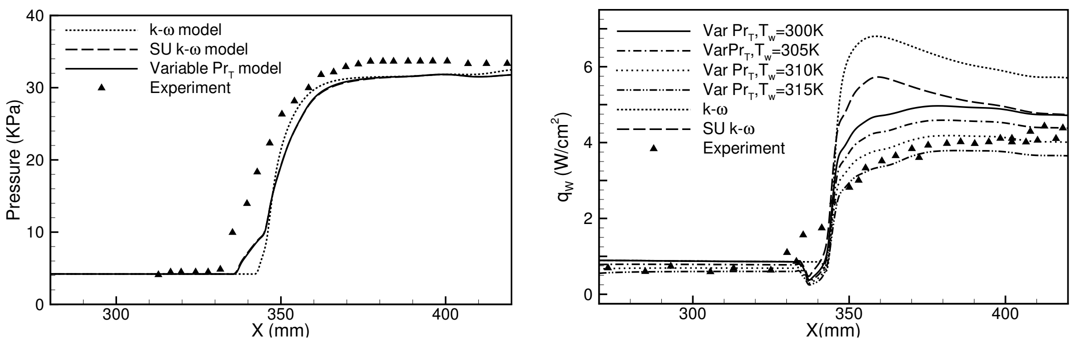

Fig. 5 Comparison of (a) surface pressure and (b) wall heat flux for $\beta = 10^o$ and $M_\infty = 5$ using standard k-ω, shock-unsteadiness modified k-ω and variable Pr_T models with the experimental data of Schulein [24].

separation caused by the weak incident shock wave. The adverse pressure gradient magnitude is high across the shock wave but reduces sharply as the boundary layer develops downstream of the reattachment point. The reduction in Pr_T follows this trend. There is a large effect of the lower Pr_T in the vicinity of the shock waves (x $\simeq$ 340 mm), and the surface heat flux is significantly reduced compared to the standard and SU k-ω models (Fig. 7). Further downstream, the variable Pr_T results approach the SU k-ω model, and therefore over-predict the experimental data.

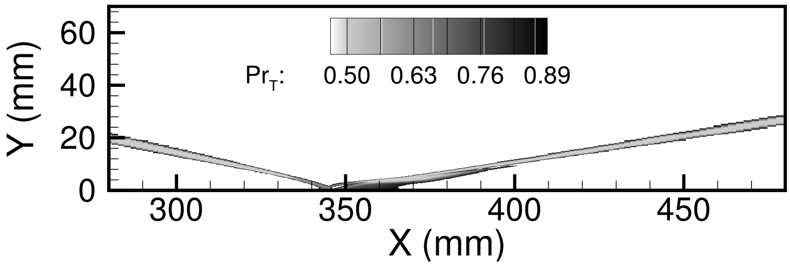

Fig. 6 Distribution for the turbulent Prandtl number computed for the 6 degree SBLI case using the variable Pr_T model.

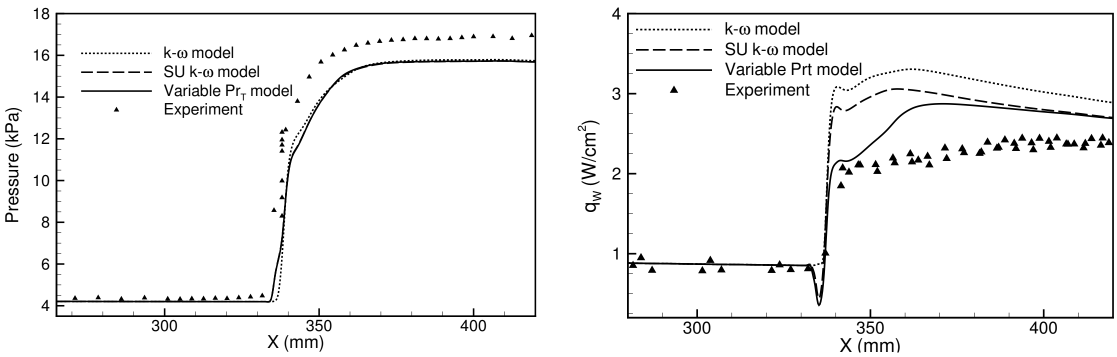

Fig. 7 Comparison of (a) surface pressure and (b) wall heat flux for $\beta = 6^o$ and $M_\infty = 5$ using standard k-ω, shock-unsteadiness modified k-ω and variable Pr_T models with the experimental data of Schulein [28].

Parametric study

The variable Pr_T model in Eq. (5) is formulated using several empirical parameters. The model predictions are found to be most sensitive to the value of the exponent (1/8). Simulations of the

14 and 10 deg. SBLI cases using different values of the exponent, in the range of 1/16 to 1/4, are presented in Fig. 8. The results show that a higher (lower) value of the exponent will increase (decrease) the peak heat flux value. Also, the lower surface heat flux obtained using a lower value of Pr_T (exponent = 1/16) is closer to the experimental data. There is a noticeable improvement in the heat flux prediction in the 10 deg case and the same can be expected for the 6 deg interaction.

The other parameters in Eq. (5) are found to have minimal effect on the computed surface heat flux. In particular, varying the coefficient (1/3) makes no noticeable difference, and the results are identical to those presented above. As expected, the surface pressure and skin friction coefficients are insensitive to the variations in the Pr_T values. The value 0.89 for zero pressure gradient boundary layers corresponds to the traditionally accepted value of the turbulent Prandtl number. On the other hand, the adverse pressure gradient function uses the boundary layer thickness δ_0 upstream of the shock interaction as the characteristic length scale. Changing the value of δ_0 by ± 25% is found to have negligible effect on the numerical predictions.

Comparison with other models

Figure 9 presents the surface heat flux computed using three algebraic variable Pr_T models. The model by Kays [10] prescribes Pr_T as a function of the ratio ν_T/ν as per Eq. (1) in the viscous sub-layer and the log layer of a zero pressure gradient boundary layer. We apply the model in the

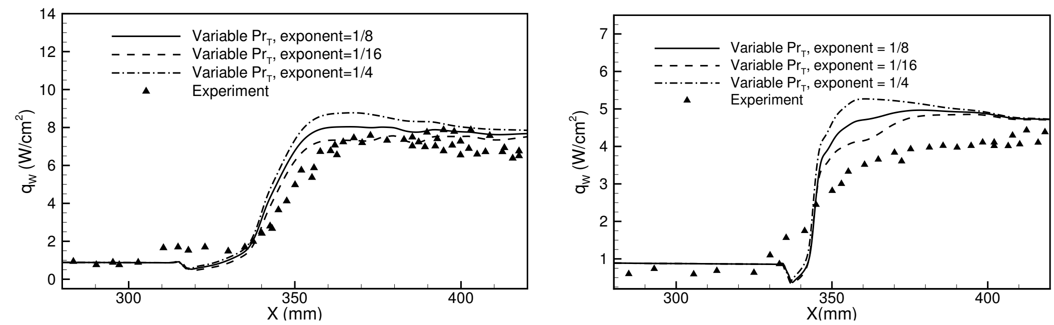

Fig. 8 Comparison of wall heat flux with different exponent values for (a) $\beta = 14^o$ and $M_\infty = 5$ and (b) $\beta = 10^o$ and $M_\infty = 5$ using variable Pr_T models with the experimental data of Schulein [24].

entire boundary layer, as changes in Pr_T in the outer part of the boundary layer do not affect the surface heat flux in the current test cases. In a similar way, we use Blackwell's model [11] given by Eq. (2) to compute Pr_T as a function of ν_T/ν, the streamwise velocity gradient, and boundary layer integral parameters. We restrict the model to regions outside of the recirculation bubble, where Re_θ and the shape factor H are not well defined. The heat flux predictions (Fig. 9a) show that the current model gives the lowest, and therefore the closest to the experimental data, in the 10 deg SBLI case. This is true in the reattachment region (345 mm < x < 380 mm), and the three model predictions are comparable further downstream. The differences between the three models is smaller in the 14 deg. interaction (Fig. 9b), with Kays model predictions comparable to the current model and Blackwell's model marginally overpredicting the data.

Figure 10 compares the current model with the differential equation based variable Pr_T model proposed by Xiao et al. [12] specifically for SBLI flows. Heat flux results for the 10 and 14 deg. SBLI cases are reproduced from the original reference, where Pr_T is computed by solving additional modeled transport equations for enthalpy variance and the dissipation rate of the enthalpy variance. The data shows that the current algebraic model for Pr_T predicts lower surface heat flux than the variable Pr_T model of Xiao et al [12]. The difference is large for the 14 deg interaction (Fig. 10b), and the current model is a significant improvement over existing variable Pr_T models for SBLI applications. The current model is also far easier to implement in an existing CFD code than

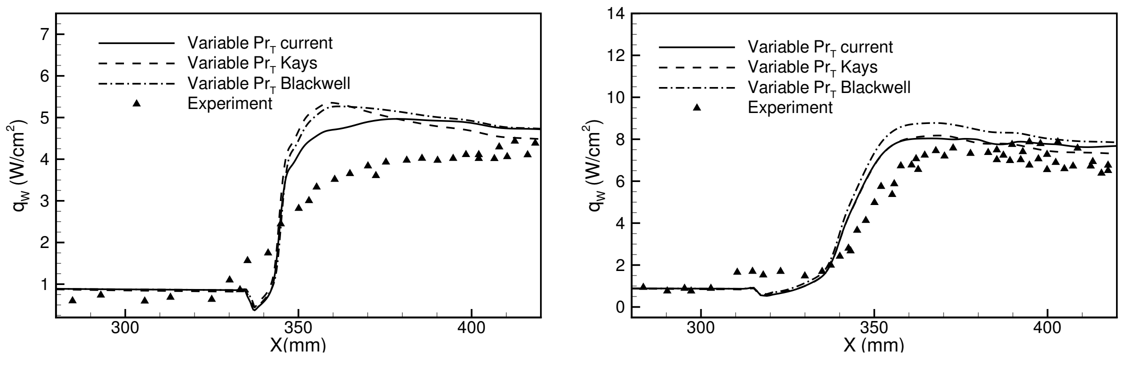

Fig. 9 Comparison of wall heat flux with different variable Pr_T models for (a) $\beta = 10^o$ and $M_\infty = 5$ and (b) $\beta = 14^o$ and $M_\infty = 5$ with the experimental data of Schulein [24].

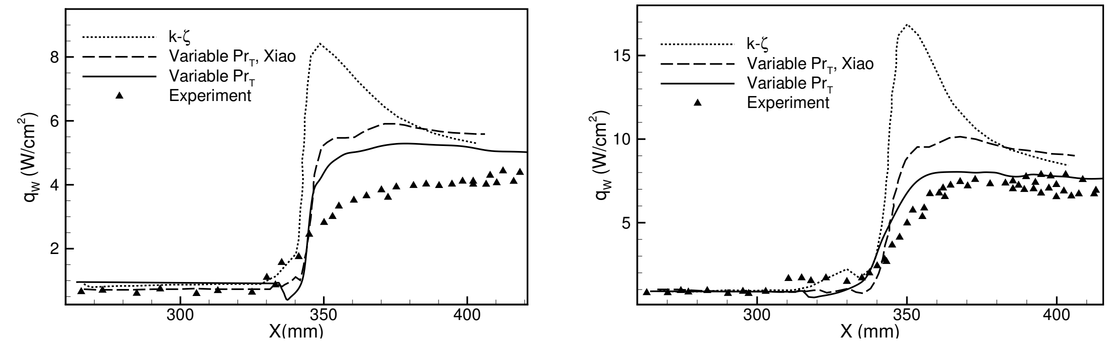

Fig. 10 Comparison of wall heat flux between Xiao et al. [12] and current variable Pr_T model for (a) $\beta = 10^o$ and $M_\infty = 5$ and (b) $\beta = 14^o$ and $M_\infty = 5$ with the experimental data of Schulein [24]. The heat flux result with k-ζ model is also reported in the plot.

differential equation based models. Even the algebraic form used in the current formulation is simpler than other algebraic models, which rely on local boundary layer integral parameters. The present model is therefore more attractive for computing complex flow configurations.

Supersonic SBLI case

We next consider a Mach 3.44 oblique shock impingement SBLI, for which experimental data is provided in Ref. [29]. The shock generator angle is 7.8 deg. and the oblique shock impinges on a flat plate boundary layer with unit Reynolds number of 8.9×10^6/m. The boundary layer thickness upstream of the interaction ($\delta_0 = 1.65$ cm) is specified, and inlet profile is generated to match this value. Isothermal wall temperature of 0.44 times the total temperature is specified in the experimental data and the shock impingement point of 250 mm from the leading edge of the plate is matched in the simulation.

We compute the SBLI flow using a 300×350 computational grid, based on the grid refinement study of the previous cases, and the results are shown in Fig. 11. Skin friction data shows a finite, but small separation bubble near the shock impingement point, and the SU k-ω model, with and without the variable Pr_T modification, is able to reproduce it correctly. The skin friction data before separation and after reattachement is also predicted well. By comparison, the standard k-ω model suppresses flow separation in this case, although there is a dip in the computed skin friction.

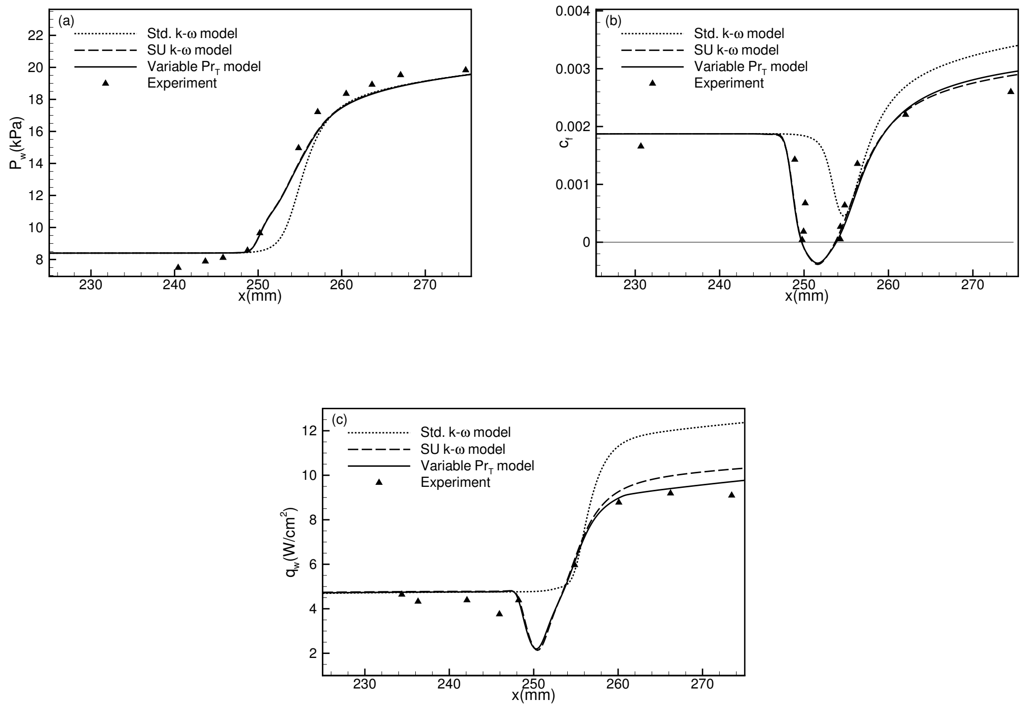

Fig. 11 Comparison of (a) surface pressure, (b) skin friction coefficient and (c) wall heat flux for $\beta = 7.8^o$ and $M_\infty = 3.44$ using standard k-ω, shock-unsteadiness modified k-ω and variable Pr_T models with the experimental data of Back [29].

Heat transfer predictions by the current variable Pr_T model are in excellent match with the experimental data, especially at reattachment and in the recovering boundary layer. The SU k-ω model is marginally higher, whereas the standard k-ω model overpredicts the experimental measurements by about 30 %. Surface pressure shows an increase at the separation point, and it is once again captured well by the SU k-ω and variable Pr_T models. The standard k-ω model gives a delayed pressure rise, due to the absence of the upstream influence of the separation bubble.

Hyperpersonic SBLI case

Finally, we compute a Mach 11.3 flow over a 36 deg. compression corner [30]. The flow unit Reynolds number is 36×10^6/m and total enthalpy is 1.63 MJ/kg. The plate length is 1.02 m, followed by a 0.3 m ramp. Freestream and wall temperatures are specified as 61 K and 300 K respectively, and the freestream density is 0.082 kg/m^3. Figure 12a shows the pressure distribution in the vicinity of the comrpession corner. We note that the surface pressure is plotted on logarithmic scale so as to enhance the separation point pressure rise at about x = 0.95 m. The standard k-ω model predicts attached flow, whereas the shock-unsteadiness correction gives a separation bubble at the corner; the computed separation point is upstream of the experiment. The flow topology is similar to that presented in [17], where an intersection of the separation shock with the reattachment shock results in a Type VI shock-shock interaction. A local peak in surface pressure is observed (around x = 1.1 m) followed by a sharp decrease due to the expansion fan generated at the triple point. See Ref. [17] for further details. The heat transfer data also shows a local peak near reattachment and the SU k-ω model, with and without variable Pr_T modification, gives the best comparison with the experimental data. Skin friction predictions are much lower than the measurements, for all three turbulence models.

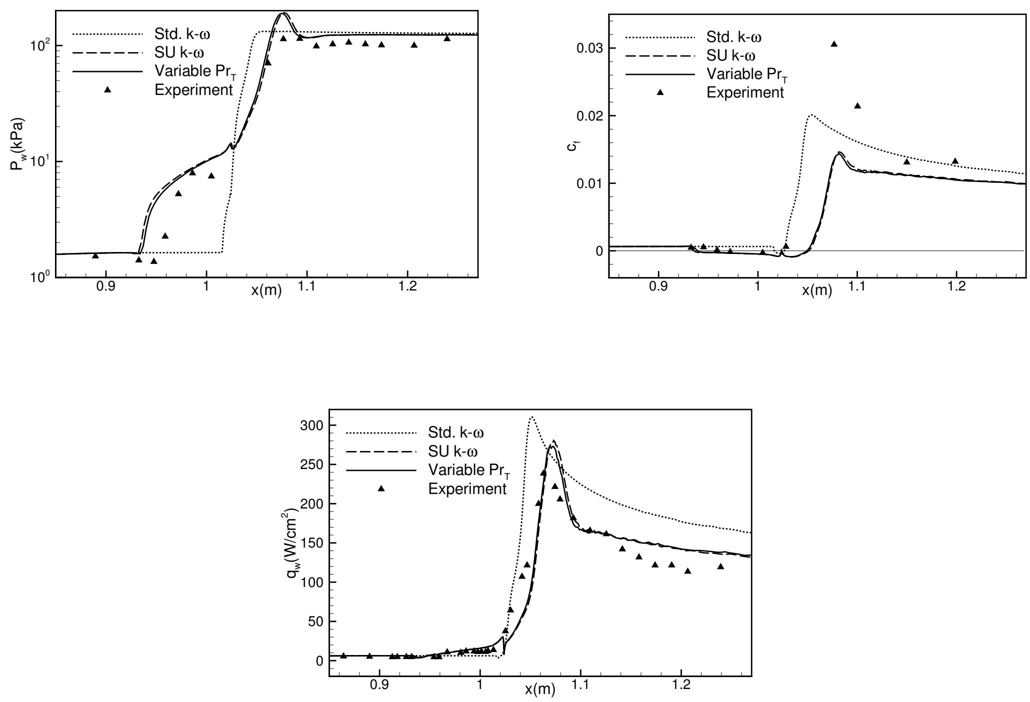

Fig. 12 Comparison of (a) surface pressure, (b) skin friction coefficient and (c) wall heat flux for $\beta = 36^o$ and $M_\infty = 11.3$ using standard k-ω, shock-unsteadiness modified k-ω and variable Pr_T models with the experimental data of Holden [30].

Conclusion

In this paper, we propose an algebraic variable turbulent Prandtl number model to accurately predict the surface heat transfer rate in shock-boundary layer interaction flows. Conventional models with constant turbulent Prandtl number, $Pr_T = 0.89$, over-predict the peak heat transfer in the interaction region. The new model is developed using phenomenological approach, and is based on experimental data for turbulent boundary layers subjected to adverse pressure gradient. We propose an algebraic formulation, which identifies regions of strong adverse pressure gradient, and lowers the turbulent Prandtl number locally. Elsewhere, it reverts back to the conventional value of 0.89 for the turbulent Prandtl number.

The variable Pr_T model is applied to oblique shocks impinging on turbulent boundary and compression corner SBLI flows. Cases with varying shock strength and Mach numbers ranging from supersonic to hypersonic values are computed, and the results are compared with the experimental measurements. The low values of the turbulent Prandtl number in the adverse pressure gradient regions significantly reduces the surface heat flux. The experimental data is reproduced well for SBLI cases with large separation, in terms of the peak heat transfer rate at flow reattachment and the streamwise variation of the surface heat flux in the recovering boundary layer. The computed results are significantly better than existing differential equation based variable Pr_T models for SBLI application. The current model adds negligible cost to computation and can be implemented in the existing CFD codes with ease.

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