

Anisotropic SST turbulence model for shock-boundary layer interaction

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ABSTRACT

Menter SST $k-\omega$ is a Reynolds-averaged Navier–Stokes based two-equation turbulence model routinely used in industry for predicting aerodynamic flows. It shows excellent performance for low-speed flows, but gives inconsistent predictions for high-speed shock-induced separated flows. The model assumption of using a constant value of 0.31 for the structure parameter contradicts experimental observations. The model is also unable to predict Reynolds stress anisotropy generated by shock waves. In this work, we augment the SST model with quadratic eddy viscosity formulation of an explicit algebraic Reynolds stress model. A new relation for the structure parameter is proposed, making it a function of the local strain-rates and is no longer a constant in the regions of shock/turbulent boundary layer interaction (SBLI). Additional shock-physics is introduced using (Sinha et al., 2003) shock-unsteadiness model and an upper limit to the value of structure parameter is set in regions of shock waves. The new model, termed as SUQ-SST, is validated using a number of SBLI test cases ranging from supersonic to hypersonic speeds and near-incipient to fully-separated flows. Results show that the modifications do not alter the boundary layer prediction capability of the SST model. On the other hand, the new model gives significant improvement in predicting Reynolds stress anisotropy, flow separation, and surface properties in a wide range of SBLI flows.

1. Introduction

Interaction of shock waves with turbulence is important in many technological applications such as the high-speed flight of aerospace vehicles [1–4] and Inertial Confinement Fusion in energy research [5, 6]. Such phenomenon in the form of shock/boundary layer interaction (SBLI) can be found over aircraft wings, deflected control surfaces, compressor passages, around turbomachinery blades, wing-body/fan-body junctions, inside air-breathing engine inlet ducts, and rocket nozzles. SBLIs can have a detrimental effect on the working of the high-speed vehicle, particularly because of shock-induced boundary layer separation and reattachment. For example, the separation bubble in the isolator section of the scramjet engine can act as a blockage to the incoming air and may lead to engine unstart. SBLIs are generally associated with significantly high thermo-mechanical loading, especially near the flow reattachment location, and thus can result in overall vehicle failure. Thus, accurate prediction of SBLI is imperative for the design of high-speed vehicles. Engineering predictions of such flows are generally done using Reynolds-averaged Navier–Stokes (RANS) equations along with an appropriate turbulence model for closure [7–10]. The choice of turbulence model plays a significant role in determining the accuracy of the results.

Generally, linear eddy-viscosity based two-equation turbulence models are used for realistic applications. Over the years, the most

widely used and popular turbulence model is the one by Menter [11]. The SST $k-\omega$ model utilizes the desired features of standard $k-\omega$ model near the wall in a boundary layer and standard $k-\epsilon$ everywhere else in the flowfield. Thus, SST can be regarded as a zonal model. However, in contrast to the classical zonal approach, it does not require an a priori knowledge of the flowfield in order to define zonal boundaries, where different models are to be used. The switching between different sub-models is achieved by ‘smart’ functions that can distinguish between the different zones.

Menter’s analysis of the standard two-equation turbulence models shows that the classical formulation of the eddy viscosity μ_T violates Bradshaw’s relation that the principal turbulent shear stress is proportional to the turbulence kinetic energy (TKE) in a boundary layer [12]. This shortcoming is attributed to the failure of standard two-equation models in case of adverse pressure gradient boundary layers. Menter modified the definition of eddy-viscosity so as to enforce Bradshaw’s relation. A modification to μ_T that forces the principal turbulent shear stress to be proportional to TKE simulates the transport effect of the principal shear stress. Hence, the resulting model is called shear stress transport (SST) model.

The standard SST $k-\omega$ model gives excellent predictions for incompressible as well as low-speed subsonic and transonic flows with

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adverse pressure gradient and separation [11–14]. For the transonic axisymmetric bump geometry, surface pressure distribution along with shock location predicted by the model matches well with experiment for free-stream Mach numbers of 0.85–0.925 [14]. However, the model significantly overpredicts the separation in case of Mach 2.3 supersonic impinging shock case with separated flow [15,16]. The model is also shown to incorrectly predict the Reynolds stresses, especially the anisotropy in normal Reynolds stresses, owing to the use of isotropic Boussinesq hypothesis [16].

Application of the SST model to Mach 2.9 supersonic impinging shock cases with varying strengths of SBLIs in terms of the flow deflection angle results in significantly bigger separation than the experiments for the separated flow cases [14]. Similarly, the model predicts almost twice the size of separation than the experiments for Mach 2.8 supersonic compression corner cases for moderately to fully separated flows [14]. Also, for SBLI caused by a sonic underexpanded jet injected into a Mach 3.75 supersonic cross flow, the resulting shock-induced separation ahead of the jet injection slot over the flat plate is significantly overpredicted by the model [17]. The discrepancy in the results is found to increase with the increase in jet exit to ambient pressure ratio. In general, SST model is found to overpredict the separation extent in case of supersonic SBLI flows and thus results in an incorrect surface pressure and skin friction distribution.

On the other hand, the SST model significantly underpredicts the separation for Mach 8, 33° compression corner SBLI [18]. The model results also show a significant underprediction of separation for 38° compression ramp case with a free stream of Mach 9.22 [10]. Similar underpredictions are also obtained for cone-flare and compression ramp geometries with fully separated flows for a free stream of Mach 11 [19, 20]. However, for the HiFire I configuration with Mach 7.9 freestream and a 33° flare, SST model significantly overpredicts the size of the separated region at the cylinder/flare junction [21]. Thus, standard SST model gives inconsistent predictions for the shock-induced separation for the hypersonic SBLI flows.

A number of variants of the standard SST model are available in the literature (see Appendix D). Brown [18] computes a series of hypersonic SBLI cases with a modified version of the standard SST model. The computations show that the model slightly overpredicts the separation for Mach 5 impinging SBLI with 10° shock generator, while it underpredicts the separation for 14° case. Model underpredictions for the separation size are also observed for Mach 8.2 impinging shock with 10° shock generator. Overall, the standard SST model along with its variants (as discussed in Appendix D) are found to give inaccurate predictions for separation size and location along with surface pressure and skin friction distribution for supersonic and hypersonic SBLI cases.

The SST model assumes a constant value of principal Reynolds shear stress over turbulence kinetic energy and the ratio is called as the structure parameter (a_1). The assumption is based on incompressible flat plate turbulent boundary layer in equilibrium [22–25]. Interestingly, even in the case of compressible attached flows, the structure parameter is found to be near to the SST model's standard value of $a_1 = 0.31$ for most parts of the boundary layer including the log layer [26,27]. However, for high-speed flows with shock-induced adverse pressure gradients, experimental evidence suggest otherwise [27–31]. In fact, experiments show that the value of structure parameter increases in the regions of SBLI. Thus, attempts to modify the value of structure parameter in an ad-hoc manner to improve the separation predictions of the SST model can be found in the literature [15,32–34].

Another major limitation of the SST model is that it employs a linear relation between the Reynolds stress and the mean strain rate tensor, known as Boussinesq hypothesis. Linear eddy viscosity models do not represent correct turbulence physics in the sense that they are not able to predict anisotropy in normal Reynolds stresses and the effects arising because of it [35]. Some applications where Boussinesq hypothesis fails are : flows with sudden changes in mean strain rate, flows over curved surfaces, flows in ducts with secondary motions, and

three-dimensional flows [36]. Reynolds stress anisotropy is a key feature of shock-turbulence interaction [37,38]. A shock wave has strong directionality, i.e. it alters the flow quantities differently between the shock-normal and shock-transverse directions. The anisotropy in the post-shock turbulence is also a function of Mach number, and traditional two-equation models based on linear eddy-viscosity formulation cannot predict it [39].

Strictly speaking, turbulence physics is better represented by Reynolds stress model (RSM), but it requires the solution of additional strongly-coupled equations as against the two less-strongly coupled equations in case of the SST model. Incorporating shock-physics into Reynolds stress models can lead to additional complexity in terms of the model parameters being function of the local shock orientation [39]. Thus, several efforts try to develop computationally more feasible approaches to include additional physics in the eddy viscosity models [40–47]. Such attempts mainly focus on using a non-linear relationship between the Reynolds stresses and the mean strain rate and vorticity tensors.

A number of non-linear eddy viscosity models, derived along quite different routes, are available in the literature with varying levels of complexities in terms of their formulation and numerical implementation [48]. Such models are developed to improve the predictions of Reynolds stress fields in a given flow and employ either quadratic, cubic, or higher order expansions of the Reynolds stresses in terms of the mean strain-rate and vorticity tensors. Also, the turbulent eddy viscosity is sensitized to the mean strain and vorticity tensors. However, majority of such models are developed for incompressible flows and a few are explored to predict transonic separated flows [43,44,47,49, 50]. A limited amount of work has been directed towards predicting supersonic and hypersonic SBLIs using such models [51,52].

Generally, a quadratic expansion of the Reynolds stresses is found to be sufficient to predict anisotropy in normal Reynolds stresses [48]. Again, a number of choices are available, however, for the purpose of this work, the Explicit Algebraic Reynolds Stress Model (EARSM) by Rung et al. [49] is considered. This EARSM is an improved version of the EARSM by Gatski and Speziale [53] and is developed for compressible flows. It enjoys the desirable properties inherent to the elaborate second-moment closure of Speziale, Sarkar, and Gatski [54]. It promises to be an application oriented alternative and offers a viable approach towards an all-speed engineering turbulence modelling framework. The simplicity of this EARSM formulation, especially for three-dimensional mean flows, makes the CFD implementation easy. The model has been implemented in the DLR-TAU and DLR-Flower codes, however the model application has been restricted to predicting transonic separated flows in the past [49,50].

In this work, we attempt to extend the applicability and improve the range of validity of the linear eddy-viscosity based SST model. The main objective is to develop a pragmatic engineering model which is able to accurately predict the distribution of surface pressure and skin friction coefficient for a range of high-speed SBLI flow conditions and geometric configurations. The major focus is the correct prediction of shock-induced separation location and the extent of the separation bubble, as judged from the wall pressure and skin friction distribution. The model development strategy followed here aims to include additional physics to the SST model by introducing minimum possible modelling complexities and without paying a large penalty on speed and robustness. The model is expected to give improved predictions of the Reynolds stress anisotropy and a correct variation of the structure parameter in the SBLI regions.

Secondly, in order to represent the correct physics of the behaviour of structure parameter in SBLI flows, we propose a new relation for the structure parameter making it a function of the local mean velocity gradients. This is based on a non-linear relationship between the Reynolds stress and strain-rate and vorticity tensors, thus transcending the Boussinesq hypothesis used in the standard SST model. Additional

shock-physics is introduced in the model by modifying the eddy viscosity at a shock wave using the physics-based shock-unsteadiness (SU) model of Sinha et al. [55]. The SU model brings in the physical effect of unsteady shock oscillations on turbulence passing through a shock wave and gives an upper limit to the values of structure parameter in the shock regions.

Important thing to note here is that the model development strategy followed in this work is not limited to the EARSM by Rung et al. [49] only. It can be readily used with other non-linear models or EARSMs available in the literature. For example, another EARSM developed for compressible flows is by Wallin and Johansson (WJ EARSM) [56]. The model is based on a recalibrated Launder, Reece and Rodi Reynolds stress transport model [57], and consists of a quartic expansion of the Reynolds stresses for a general three-dimensional mean flow. It has been previously applied to hypersonic impinging shock/turbulent boundary layer interaction and results in an underprediction of separation bubble size for moderately to fully separated flows [51]. The implication of using other non-linear model/EARSM is discussed in Section 2.6.

The paper is organized as follows. Section 2 gives the development details of the new model. First, Menter SST k - ω model equations are presented, followed by the quadratic eddy viscosity formulation of Rung et al. [49]. Modifications to the structure parameter and eddy viscosity in a turbulent boundary layer and at a shock wave in the quadratic eddy viscosity modelling framework are discussed. Simulation methodology is explained in Section 3 with details of the test cases used for model validation. Finally, model predictions are compared to available experimental data for a series of SBLI cases in Section 4.

2. Model development

In this section, we start with the SST model of Menter and propose modifications to include non-linear Reynolds stress-strain relation and shock-physics based correction applicable to SBLI flows.

2.1. Reynolds-averaged Navier–Stokes equations

For compressible flow, the Reynolds-averaged Navier–Stokes equations are given by [36],

$$\frac{\partial \bar{\rho}}{\partial t} + \frac{\partial}{\partial x_i}(\bar{\rho} \bar{u}_i) = 0 \quad (1)$$

$$\frac{\partial}{\partial t}(\bar{\rho} \bar{u}_i) + \frac{\partial}{\partial x_j}(\bar{\rho} \bar{u}_j \bar{u}_i) + \frac{\partial(-\tau_{ij})}{\partial x_j} = -\frac{\partial \bar{p}}{\partial x_i} + \frac{\partial \sigma_{ij}}{\partial x_j} \quad (2)$$

$$\begin{aligned} & \frac{\partial \bar{\rho} \bar{E}}{\partial t} + \frac{\partial}{\partial x_j}(\bar{\rho} \bar{H} \bar{u}_j) + \frac{\partial(-\tau_{ij}) \bar{u}_i}{\partial x_j} \\ & = \frac{\partial}{\partial x_j} \left[\frac{\bar{\mu}}{Pr} \frac{\partial \bar{h}}{\partial x_j} + \frac{\mu_T}{Pr_T} \frac{\partial \bar{h}}{\partial x_j} + \left(\bar{\mu} + \frac{\mu_T}{\sigma_k} \right) \frac{\partial k}{\partial x_j} \right] + \frac{\partial(\sigma_{ij} \bar{u}_i)}{\partial x_j} \end{aligned} \quad (3)$$

where the overbar and tilde represent Reynolds and Favre-averaged quantities, respectively, and repeated indices indicate Einstein summation. Here, ρ is the density, p is the pressure, T is the temperature, and u_i is the component of velocity in the x_i direction. The underlined terms in the set of RANS equations describe the average effect of turbulence on the mean flow. Notably, the gradients of Reynolds stresses in the momentum equation (2) play a significant role in determining the separation characteristics like separation location and length of the separation bubble in case of flows involving shock-induced separations.

The averaged viscous stresses of a Newtonian fluid are described by,

$$\sigma_{ij} = 2\bar{\mu}S_{ij} = \bar{\mu} \left(\frac{\partial \bar{u}_i}{\partial x_j} + \frac{\partial \bar{u}_j}{\partial x_i} - \frac{2}{3} \frac{\partial \bar{u}_k}{\partial x_k} \delta_{ij} \right) \quad (4)$$

The averaged molecular viscosity is provided by Sutherland law,

$$\bar{\mu} = 1.458 \times 10^{-6} \frac{\bar{T}^{3/2}}{\bar{T} + 110.3} \quad (5)$$

In Eq. (3), we made use of Fourier type law for modelling both laminar and turbulent heat-flux vectors as per Wilcox [36] with $Pr = 0.72$, $Pr_T = 0.89$ and μ_T is the eddy viscosity. An ideal gas is assumed, $\bar{p} = \bar{\rho} R \bar{T}$, where the specific gas constant for air is $R = 287.1$ J/kg K, and the mass-averaged specific total energy and total enthalpy are given by,

$$\begin{aligned} \bar{E} &= \bar{e} + \frac{1}{2} \bar{u}_i \bar{u}_i + k \\ \bar{H} &= \bar{E} + \frac{\bar{p}}{\bar{\rho}} = \bar{h} + \frac{1}{2} \bar{u}_i \bar{u}_i + k \end{aligned} \quad (6)$$

Assuming a perfect gas, the specific heats are constant, so that the averaged specific internal energy and enthalpy are given by,

$$\bar{e} = C_V \bar{T} \quad \bar{h} = C_p \bar{T} \quad (7)$$

where, $\gamma = C_p/C_V = 1.4$ for air. The Reynolds stress tensor τ_{ij} is generally related to the mean flow velocity gradients either linearly using Boussinesq hypothesis as in Menter SST k - ω model or using a non-linear relation as in an EARSM.

2.2. Menter SST k - ω model

Menter SST is a hybrid model based on the k - ω framework, which switches between high-Re k - ϵ formulation in free-shear layers and Wilcox 1988 k - ω model in the boundary layer regions near a wall. The switching is done using a function F_1 and the model equations are given by [11],

$$\bar{\rho} \frac{\partial k}{\partial t} + \bar{\rho} \bar{u}_j \frac{\partial k}{\partial x_j} = \tau_{ij} \frac{\partial \bar{u}_i}{\partial x_j} - \beta^* \bar{\rho} k \omega + \frac{\partial}{\partial x_j} \left[(\mu + \mu_T \sigma_k) \frac{\partial k}{\partial x_j} \right] \quad (8)$$

$$\begin{aligned} \bar{\rho} \frac{\partial \omega}{\partial t} + \bar{\rho} \bar{u}_j \frac{\partial \omega}{\partial x_j} &= \frac{\gamma}{v_T} \tau_{ij} \frac{\partial \bar{u}_i}{\partial x_j} - \beta \bar{\rho} \omega^2 + \frac{\partial}{\partial x_j} \left[(\mu + \mu_T \sigma_\omega) \frac{\partial \omega}{\partial x_j} \right] \\ &+ 2(1 - F_1) \frac{\bar{\rho} \sigma_{\omega_2}}{\omega} \frac{\partial k}{\partial x_j} \frac{\partial \omega}{\partial x_j} \end{aligned} \quad (9)$$

where, k , ω , and v_T are the turbulence kinetic energy (TKE), specific turbulence dissipation rate, and kinematic eddy viscosity, respectively. The model constants and the blending functions are given in Appendix A.

Reynolds stress components τ_{ij} are linearly related to the mean strain-rates using Boussinesq approximation,

$$\tau_{ij} = 2\mu_T S_{ij} - \frac{2}{3} \rho k \delta_{ij} \quad (10)$$

Eddy viscosity for this model is given by,

$$\mu_T = \frac{\bar{\rho} a_1 k}{\max(a_1 \omega, \Omega F_2)} = \begin{cases} \bar{\rho} k / \omega & \text{Standard form} \\ \bar{\rho} a_1 k / \Omega & \text{SST modified form} \end{cases} \quad (11)$$

The function F_2 switches eddy viscosity smoothly between the SST modified form active inside boundary layers and the standard form everywhere else. Here, Ω is the vorticity magnitude, while some variants of the SST (given in Appendix C) use the strain-rate magnitude S instead. Here, $\Omega = \sqrt{-2II_\Omega}$ and $S = \sqrt{2II_S}$, with the second invariants of vorticity and strain-rate tensors given by $II_\Omega = \Omega_{ij} \Omega_{ji}$ and $II_S = S_{ij} S_{ji}$, respectively. The vorticity and strain-rate tensors are $\Omega_{ij} = (1/2)(\partial \bar{u}_i / \partial x_j - \partial \bar{u}_j / \partial x_i)$ and $S_{ij} = (1/2)(\partial \bar{u}_i / \partial x_j + \partial \bar{u}_j / \partial x_i) - (1/3)(\partial \bar{u}_k / \partial x_k) \delta_{ij}$, respectively.

2.3. Quadratic eddy viscosity formulation

EARSOM of Rung et al. [49] is a realizable quadratic eddy viscosity model and is based on the original EARSOM by Gatski and Speziale [53]. Following Wallin and Johansson [56], a general relationship between Reynolds stress, mean strain-rate and vorticity tensors can be written in terms of an effective turbulent viscosity μ_T and an “extra” (ex) anisotropy term $a_{ij}^{(ex)}$ as,

$$\tau_{ij} = 2\mu_T S_{ij} - \frac{2}{3}\bar{\rho}k\delta_{ij} - \bar{\rho}ka_{ij}^{(ex)} \quad (12)$$

The extra anisotropy tensor is given by a quadratic form [49],

$$a_{ij}^{(ex)} = -2C_\mu^* \tau^2 \left[\beta'_2 (S_{ik}\Omega_{kj} - \Omega_{ik}S_{kj}) - \beta'_3 (S_{ik}S_{kj} - \frac{1}{3}S_{kl}S_{kl}\delta_{ij}) \right] \quad (13)$$

where $\tau = 1/(C_\mu\omega)$ is the turbulence time scale and $C_\mu = 0.09$. Eddy-viscosity for this EARSOM is similar to the standard form of μ_T for a linear eddy viscosity based $k-\omega$ model in Eq. (11) with an additional factor of C_μ^*/C_μ multiplied to it,

$$\mu_T = \frac{C_\mu^* \bar{\rho}k}{C_\mu \omega} \quad (14)$$

where C_μ^* is the anisotropy parameter defined as,

$$C_\mu^* = \frac{\beta'_1}{1 - \frac{2}{3}\eta^2 + 2\xi^2} \quad (15)$$

with the β' coefficients and the auxiliary functions given by,

$$\begin{aligned} \beta'_1 &= \frac{\frac{4}{3} - C_2}{2g}, \quad \beta'_2 = \frac{2 - C_4}{2g}, \quad \beta'_3 = \frac{2 - C_3}{g} \\ \eta^2 &= \frac{\beta_3'^2 \hat{S}^2}{8}, \quad \xi^2 = \frac{\beta_2'^2 \hat{\Omega}^2}{2}, \quad g = g_{ic} + g_c \\ g_{ic} &= f_g(C_1 - 1) + \psi, \quad f_g = 1 + 0.95 \left(1 - \tanh \left(\frac{\hat{S}^2}{4.6225} \right) \right) \\ \psi &= \frac{\hat{S}^2}{4 + 1.83\sqrt{0.8\hat{\Omega}^2 + 0.2\hat{S}^2}} \\ g_c &= -0.15M_t \left[\frac{P}{(\bar{\rho}e)} - \frac{4}{3}M_t \right], \quad \frac{P}{\bar{\rho}e} = \frac{0.8 + \psi}{1.9} \end{aligned} \quad (16)$$

The non-dimensional strain-rate and vorticity magnitudes are $\hat{S} = \tau S$ and $\hat{\Omega} = \tau\Omega$, respectively, and the model constants are : $C_1 = 2.5$, $C_2 = 0.39$, $C_3 = 1.25$, $C_4 = 0.45$. Compressibility correction (g_c) in the form of Sarkar’s pressure-dilatation model is adopted [49]. It models the dilatational part of the pressure-strain correlation, which represents the mechanism of turbulence redistribution and relaxation. Here, $M_t = \sqrt{2k/\bar{a}}$ is the turbulent Mach number and $\bar{a}$ is the Favre-averaged speed of sound.

2.4. Modified structure parameter

Menter [11] assumes a general linear relationship for the principal Reynolds stress, $\tau_{principal} = \mu_T \Omega$, where the vorticity magnitude represents the mean velocity gradients. This is equated to Bradshaw’s assumption,

$$\tau_{principal} = \bar{\rho}a_1 k \quad (17)$$

which states that the principal turbulent stress is proportional to turbulence kinetic energy in a turbulent boundary layer, to arrive at the SST modification in Eq. (11). Here, the structure parameter a_1 assumes a constant value of 0.31.

We follow a strategy similar to Menter and assume the following quadratic relation between the principal stress and mean velocity gradient,

$$\tau_{principal} = \mu_T D + C_\mu^* \bar{\rho}k(\tau D)^2(\beta'_2 - \beta'_3) \quad (18)$$

Here, $D = \sqrt{(\partial\tilde{u}_i/\partial x_j)(\partial\tilde{u}_i/\partial x_j)} = \sqrt{(S'^2 + \Omega^2)/2}$ is the magnitude of deformation rate tensor $\partial\tilde{u}_i/\partial x_j$ and $S' = \sqrt{2S'_{ij}S'_{ij}}$ with $S'_{ij} = (1/2)(\partial\tilde{u}_i/\partial x_j + \partial\tilde{u}_j/\partial x_i)$. Note that, we have chosen to use the magnitude of deformation rate tensor as against the vorticity magnitude Ω used by Menter. In this way, we are retaining the physics incumbent in the vorticity magnitude in the principal stress as in Menter’s proposal. Moreover, it ensures that the dominant nature of the dilatational strain-rate is captured in the vicinity of shock waves.

Equating Eq. (18) to Bradshaw’s relation (Eq. (17)), gives us

$$a'_1 = C_\mu^* \tau D [1 - \tau D(\beta'_2 - \beta'_3)] \quad (19)$$

where we use $a_1 = C_\mu^* \tau D$ obtained from a log-layer analysis of the EARSOM equations for an equilibrium zero-pressure-gradient turbulent boundary layer (shown in Appendix B). Thus, the structure parameter for this model is a function of the mean velocity field in the form of the magnitude of the deformation-rate tensor and, unlike in the SST model, is not a constant. The anisotropy parameter C_μ^* also appears in the above equation and it responds to the changes in strain rate and vorticity magnitude.

It is interesting to note that the constant value 0.31 of the structure parameter used in SST model is based on incompressible equilibrium boundary layer data. However, it is also found to be a good approximation in adverse-pressure-gradient subsonic and transonic boundary layers. For incompressible high Reynolds number flows with Re_θ (θ being the momentum thickness) in the range of 2573 – 57720, Smits and Dussauge [25] found that a_1 lies in the range of 0.28 – 0.34. Experimental [27] and DNS [26] studies of compressible zero-pressure-gradient boundary layer also found that a_1 assumes values similar to incompressible flows. However, in case of non-equilibrium compressible flows, such as SBLIs, experiments show that a_1 is higher than 0.31 in the interaction region and the increase is significant.

Rose and Murphy [27] performed experiments for an impinging shock configuration (with a free stream Mach number around 4 and $Re_\delta = 8.7 \times 10^4$, where δ is the boundary layer thickness) to calculate a_1 . The experiments involve near incipient shock-induced separation. Measurements reveal that the structure parameter increases to a value as high as 0.44 in the interaction region. The oblique impinging/reflected shock experiments [31,58], as a part of the European Union SBLI research project referred to as UFAST-Unsteady Effects of Shock Wave Induced Separation, with Mach 2.3 flow also show that a_1 reaches a value of 0.4 in the SBLI region. Ardonneau [59] and Smits and Muck [28] make similar observations for a series of compression corner SBLI experiments. Thus, there is a definite evidence that the physics of compressible flows with adverse pressure gradient is significantly different than that of incompressible separated flows. As a matter of fact, in an incompressible boundary layer, a_1 decreases in response to adverse pressure gradient, as observed by Bradshaw [60], whereas it increases in SBLI flows [30].

2.5. Analysis at a shock wave : SU limiter

The structure parameter given in Eq. (19) is a function of the strain-rate tensor, which can take large numerical values at a shock wave. In fact, the strain rate can increase with grid refinement as $1/\Delta$, where Δ is the grid resolution at a shock wave. As a result, the structure parameter can assume unphysical values. So, we use the physics of shock-turbulence interaction to prescribe possible limiting values of the effective structure parameter at a shock wave. This follows a physical argument rather than the empirical stress limiter C_{lim} found in the literature [33]. The following analysis is based on Sinha et al. [55] shock-unsteadiness (SU) model.

It is shown that the standard form of TKE production in Eq. (8) grossly overpredicts turbulence amplification at a shock wave. The shock-unsteadiness model alters the production term based on the damping effect of an unsteady shock wave, and the results match DNS

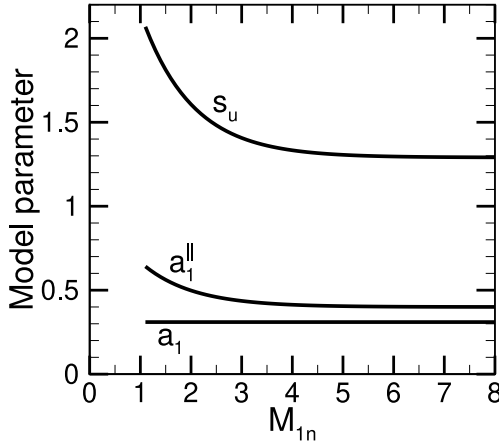


Fig. 1. Variation of the model parameters s_u and a_1'' with upstream shock-normal Mach number M_{1n} .

data for canonical shock-turbulence interaction. Shock-unsteadiness model's production term is given by

$$P_k' = -\frac{2}{3}\bar{\rho}k(1 - b_1')S_{ii} = \tau_{eq}|S_{ii}| \quad (20)$$

where S_{ii} is the mean dilatation, and b_1' is the shock unsteadiness parameter described below. The production of TKE in Eq. (20) is written in terms of the magnitude of dilatation and an equivalent Reynolds stress,

$$\tau_{eq} = (2/3)\bar{\rho}k(1 - b_1') = s_u\bar{\rho}a_1k \quad (21)$$

which brings in the effect of unsteady shock oscillations in response to incoming turbulent fluctuations. Thus, the equivalent stress τ_{eq} acts on the dilatation S_{ii} and results in the correct production of TKE at a shock wave. The equivalent stress τ_{eq} is proportional to TKE at a shock wave, with $s_u = \frac{2}{3}(1 - b_1')/a_1$. This is similar to Bradshaw's relation given in Eq. (17). However, at a shock wave the equivalent stress $\tau_{eq} > \bar{\rho}a_1k$, as $s_u > 1$. Here, the shock-unsteadiness damping parameter $b_1' = 0.4(1 - e^{1-M_{1n}})$ is a function of upstream shock-normal Mach number M_{1n} .

We now convert the equivalent stress τ_{eq} into an effective eddy viscosity to get,

$$\mu_T = \frac{\bar{\rho}a_1''k}{D} \quad \text{with} \quad a_1'' = s_u a_1 \quad (22)$$

which has a form similar to the SST modified eddy viscosity in Eq. (11), with a modified structure parameter at a shock wave. Fig. 1 shows the variation of s_u and a_1'' for a range of Mach numbers. The shock-unsteadiness model parameter s_u assumes a value of about 2 for weak shock waves and asymptotes to a value of 1.29 for $M_{1n} \rightarrow \infty$. This means that the modified structure parameter a_1'' varies from 0.62 for Mach 1 to 0.4 for hypersonic flows. We choose $a_1'' = 0.4$ as the limiting value of the structure parameter at a shock wave, so as to meet the constraint for the entire range of Mach numbers. A constant value of a_1'' makes the model implementation simple, by avoiding a model parameter that depends on the upstream Mach number.

Wilcox 2006 $k-\omega$ model uses the effective structure parameter to be 0.343 and gives better results for SBLI flows than $a_1 = 0.31$ [36]. Several numerical studies using SST model [15,32–34] also found that using a value of the structure parameter higher than its standard value of 0.31 gives improved results for SBLI flows; however all these studies increase the structure parameter in an ad-hoc manner. Our model brings in a similar effect, but in a more formal way that is based on physical analysis at a shock wave. The current model thus differs from the ad-hoc approaches found in the literature, which use empirical eddy-viscosity limiters.

2.6. A new shear stress transport model

We now combine the developments of the previous sub-sections to present the final form of the SST model for SBLI flows. There are two key developments. First, the SST model is augmented by a quadratic relation for Reynolds stresses taken from EARSMS. This leads to a modified structure parameter a_1' in Eq. (19). This is combined with the SU-modified eddy viscosity at a shock wave given in Eq. (22). We thus get an eddy viscosity model applicable to turbulent boundary layers and at shock waves,

$$\mu_T = \frac{\bar{\rho}a_1^*k}{D} = \frac{\bar{\rho}a_1k}{D(a_1/a_1^*)} \quad \text{with} \quad a_1^* = \min[a_1', a_1''] \quad (23)$$

This form is similar to the SST-modified μ_T in Eq. (11), with a_1 replaced by the effective structure parameter a_1^* .

Obviously, Eq. (23) is not desirable for the complete flowfield, since it leads to infinitely high eddy-viscosities at points where D goes to zero. So, it is supplemented with the EARSMS eddy viscosity given in Eq. (14) and rewritten here in a slightly different form as,

$$\mu_T = \frac{\bar{\rho}a_1k}{(C_\mu^*/C_\mu)a_1\omega} \quad (24)$$

To guarantee the selection of desired forms of the eddy viscosity in appropriate regions of the flowfield, we combine Eqs. (23) and (24) in the following manner,

$$\mu_T = \frac{\bar{\rho}a_1k}{\max\left[(1 - F_2')(C_\mu^*/C_\mu)a_1\omega, F_2'D(a_1/a_1^*)\right]} \quad (25)$$

similar to Eq. (11) in the original SST model. Here, F_2' is a function which identifies turbulent boundary layers and shock waves, where it assumes a value of 1, and it is identically zero in rest of the flowfield. Eq. (25) can be thought of as two different eddy-viscosity formulations applicable in two different regions in the flowfield; first term of the denominator applicable in regions where the velocity gradient magnitude goes to zero, such as the uniform freestream, and second term is applicable in the turbulent boundary layers including the adverse pressure gradient regions and at shock waves. Multiplication of the first and second terms in the denominator of Eq. (25) with $(1 - F_2')$ and F_2' , respectively ensures that relevant formulation has finite non-zero values in the appropriate regions of the flowfield as mentioned above. The “max” function then enables the choice of this pertinent formulation. Thus, Eq. (25) is formulated such that it switches to Eq. (23) inside a boundary layer and in the regions of shock waves, and reverts back to Eq. (24) elsewhere. The modified eddy viscosity given in Eq. (25) together with the quadratic Reynolds stress-strain relation given in Eqs. (12) and (13) and the $k-\omega$ equations given in Eqs. (8) and (9) is called as the SUQ-SST model.

The modelling strategy followed in this work can be easily used with other non-linear eddy viscosity model or EARSMS. For example, if EARSMS by Wallin and Johansson [56] is considered for the model development instead of EARSMS by Rung et al. [49], then the resulting changes in the model equations are discussed below. For a general three-dimensional mean flow, WJ EARSMS has a quartic expansion of the Reynolds stresses, i.e., Eq. (13) for the extra-anisotropy will have additional contributions in terms of third and fourth order powers of strain and vorticity tensors. This will then translate to a quartic relation for the structure parameter in Eq. (19) instead of a quadratic one. However, rest of the equations in the new model i.e. Eqs. (23) and (25) will have identical forms as the present model which uses EARSMS by Rung et al. [49].

The function F_2' is given by,

$$F_2' = \max[F_2, f_s] \quad (26)$$

where F_2 is the standard function used in Menter's SST model (see Appendix A). It makes sure that the SST correction is active only inside

a turbulent boundary layer. By comparison, the function f_s detects regions of shock waves and is of the form,

$$\begin{aligned} arg &= 50 \frac{S_{ii}}{\tau_s} + 5 \\ \tau_s &= \frac{\tilde{a}}{L_s}, \quad L_s = \max(\Delta, L_e), \quad L_e = \frac{0.25\sqrt{k}}{0.09\omega} \\ f_s &= \frac{1}{2} - \frac{1}{2} \tanh(arg) \end{aligned} \quad (27)$$

where, $\tilde{a}$ is the local mean speed of sound. Here, Δ is the local representative finite-volume cell size. For structured Cartesian grids, Δ can be simply calculated as $\Delta = \sqrt{\Delta x \Delta y}$, where Δx and Δy are the local grid spacings in the x and y -direction, respectively.

The function f_s smoothly transitions to 1 in shock regions from a value of zero outside shock waves. In Eq. (27), we use the mean dilatation S_{ii} normalized by a suitable time scale τ_s for shock detection. The time scale uses local representative grid size for detecting shock waves in the inviscid regions and the form is similar to that used in Bhagatwala and Lele [61]. On the other hand, for detecting shock waves penetrating inside a turbulent boundary layer, such as the separation shock foot in the impinging shock and compression corner flows described below, we use the turbulence dissipation length scale L_e to determine τ_s .

Interestingly, different forms of the function f_s have been used in previous works [62–64] for computing different configurations of SBLI flows. However, a single form of shock function given in Eq. (27), which is algebraic in nature, is found suitable to correctly capture shock waves for all the SBLI configurations considered in this work. The function f_s proposed here requires local flow data and grid details and unlike the form in Pasha and Sinha [63,64], it is independent of non-local flow properties, like the incoming boundary layer thickness, which may not be readily available.

3. Simulation methodology

The canonical SBLI flows considered in this work have two-dimensional mean flow, with minimal variation in the span-wise direction. We solve the two-dimensional Reynolds-averaged Navier–Stokes equations with Favre decomposition of the flow variables, as presented in Section 2.1. The standard SST k - ω model of Menter [11], EARSM model of Rung et al. [49], and the newly proposed SUQ-SST model are used for turbulence closure. We use a well-validated in-house CFD code that has been employed to a variety of high-speed flow simulations in the past [62–67].

The governing equations are discretized in a finite volume formulation, where the inviscid fluxes are computed using a modified, low-dissipation form of the Steger–Warming flux splitting approach [68]. The mean flow equations fully coupled with the turbulence model equations are solved as per [69]. The method is second-order accurate in stream-wise and wall-normal directions. The viscous fluxes and the turbulent source terms are evaluated using a second-order accurate central difference method. The implicit data parallel line relaxation method of [70] is used to integrate in time and reach the steady-state solution.

Following Menter [11], boundary conditions for the turbulence model equations at the wall are $k = 0$ and $\omega = 60\nu_w/\beta\Delta y_1^2$, where ν_w is the kinematic viscosity at the wall, $\beta = 3/40$ is a model constant, and Δy_1 is the distance to the next point away from the wall. The free-stream conditions used for the simulations are $\omega_\infty = 10U_\infty/L$ and $k_\infty = 0.01\nu_\infty\omega_\infty$ as per [11], where U_∞ is the free-stream mean velocity and L is a characteristic length of the domain.

3.1. Flat plate simulations

Zhang et al. [71] presents a direct numerical simulation (DNS) database of high-speed zero-pressure-gradient turbulent boundary layers developing spatially over a flat plate with nominal freestream Mach number ranging from 2.5 to 14. These DNS cases fall within the perfect gas regime. In this work, we compute the Mach 2.5 case with freestream conditions of velocity $U_\infty = 823.6$ m/s, density $\rho_\infty = 0.1$ kg/m³, and temperature $T_\infty = 270$ K. The working fluid is air with viscosity calculated using Sutherland's law. The wall is assumed to be isothermal with a wall temperature of $T_w = 568$ K. DNS provides mean properties at a location, where boundary layer thickness $\delta = 7.7$ mm, displacement thickness $\delta^* = 2.38$ mm, momentum thickness $\theta = 0.58$ mm, and the Reynolds number based on momentum thickness is $Re_\theta = 2809.9$.

A structured cartesian mesh with 400×250 cells is used for the simulations with the grid points clustered at the plate tip and exponentially stretched away from it along the wall. Exponential stretching is also used in the wall-normal direction with the distance of the first cell centre of 1×10^{-6} m from the wall. At the solid wall, a no-slip wall boundary condition is employed, whereas at the top and exit boundaries, an extrapolation condition is used.

3.2. UFAST impinging shock simulations

The experiment for this case was carried out in the Mach 2.3 supersonic wind tunnel at the Institut Universitaire des Systemes Thermiques Industriels (IUSTI) in Marseille, France [58]. The experimental data was part of the European Union SBLI research project referred to as UFAST [31]. The experiment utilizes an 8° shock generator and the incident shock would impact at $x = 336$ mm from the flat plate leading edge in the absence of a boundary layer. Mean flow profiles are provided at $x = 260$ mm from the plate leading edge, where the boundary layer thickness and displacement thickness are found to be 11 mm and 3.535 mm, respectively.

Fig. 2(a) shows the schematic of experimental set-up and the computational domain is shown using dashed lines. The freestream conditions used for the simulations are given in Table 1. The SBLI simulations start at $x = 260$ mm from the plate leading edge. Inlet conditions for the mean and turbulence variables are obtained from separate flat plate simulations and they correspond to the displacement thickness of 3.535 mm. The inlet profile for the mean flow variables is modified at the shock entry point to post-shock conditions calculated analytically (see Fig. 2(a)). For the turbulence model equations, k and ω values at the boundary layer edge in the inlet profile are prescribed as the post-shock conditions. The solid wall is considered to be adiabatic and a no-slip wall boundary condition is used. An extrapolation condition is used at the top and exit boundaries.

A structured Cartesian grid with 200×270 finite volume cells is used for the SBLI simulations. The grid size is based on a rigorous grid convergence study described in Appendix C. Grid points are clustered near the wall with a minimum spacing of 1×10^{-6} m, which is equivalent to 0.1 wall units or less along the flat plate. Clustering is also done in the interaction region where the smallest cell size along the streamwise direction is 3×10^{-4} m. An exponential stretching is used in the wall-normal direction as well as in the upstream and downstream directions of the interaction location.

3.3. Schulein's impinging shock simulations

The test case consists of a Mach 5 flow with oblique shock impinging on a turbulent boundary layer developed over a flat plate. The experiments [72] were done over a 500 mm long flat plate, and the inviscid shock was adjusted to impinge at a fixed location of 350 mm from the leading edge of the flat plate (Fig. 2(a)). The experiments provide mean flow profiles at $x = 266$ mm from the plate leading edge, where the boundary layer thickness, displacement thickness,

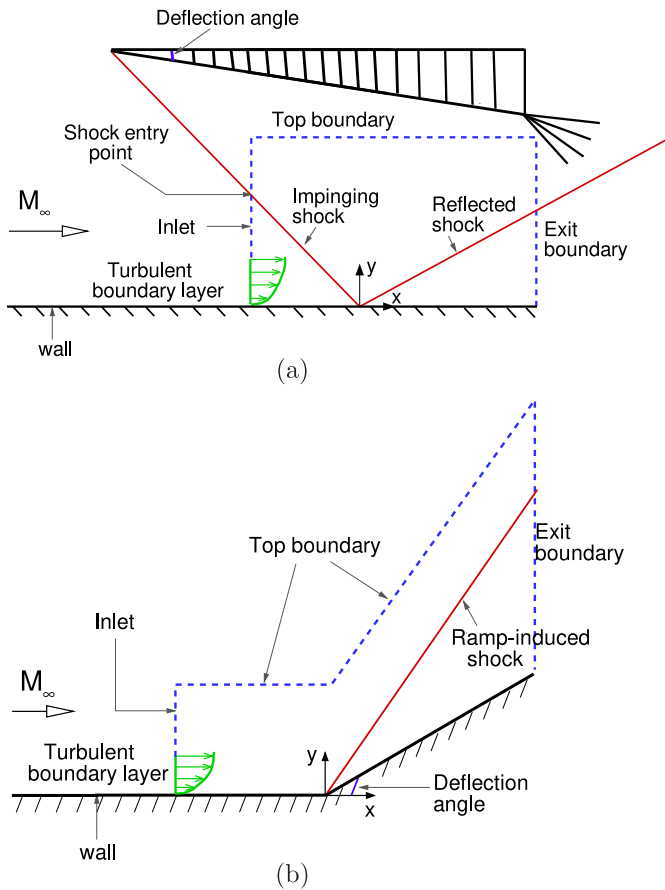


Fig. 2. Schematic of (a) incident oblique shock wave/turbulent boundary layer interaction on a flat plate and (b) two-dimensional compression corner, with the computational domain marked by dashed line for the two configurations.

Table 1
Freestream conditions for the SBLI test cases as reported in the experiments.

	Deflection angle	M_∞	ρ_∞ (kg/m ³)	T_∞ (K)	T_w (K)	$Re_\infty/m \times 10^6$	Reference
UFAST	8°	2.3	0.0966	144.6	–	5.4	[58]
Schulein	6°	5	0.2043	68.3	300	37	[72]
SBLI	10°	5	0.2043	68.3	300	37	[72]
	14°	5	0.2043	68.3	300	37	[72]
	27°	8.2	0.5090	71.1	296	147	[73]
Holden	30°	8.3	0.5060	68.9	296	149	[73]
SBLI	33°	8.1	0.4910	71.7	297	139	[73]
	36°	8.2	0.4990	70	298	145	[73]

and momentum thickness are measured as 3.81 mm, 1.576 mm, and 0.157 mm, respectively. The experimental uncertainties are quoted as approximately 2% for pressure and 4% to 10% for skin friction. The experimental data for comparison is taken from [19].

The freestream conditions for these cases are given in Table 1 and the computational domain is shown with dashed lines in Fig. 2(a). The SBLI computations begin at $x = 266$ mm from the plate leading edge. Inlet conditions are obtained from separate flat plate computations by matching the momentum thickness with the experiment. At the shock entry location (see Fig. 2(a)), inlet profile for the mean flow variables is modified and analytical relations are used to prescribe post-shock conditions corresponding to the three different flow deflection angles of 6°, 10°, and 14°. The values of k and ω at the boundary layer edge

in the inlet profile are directly used as the post-shock conditions for the turbulence variables. At the solid wall, a no-slip isothermal-wall boundary condition is applied and the wall temperature is taken to be 300 K. Mean and turbulence variables are directly extrapolated from the interior cells at the top and exit boundaries.

A structured Cartesian grid consisting of 300×400 cells is used for the SBLI simulations. The grid is exponentially stretched away from the wall and uses a near-wall spacing of 1×10^{-6} m that is equivalent to 0.55 wall units or lower along the flat plate. Exponential stretching is also used in the upstream and downstream directions of the interaction location, where the minimum cell size is 1×10^{-4} m. The grid is based on extensive grid convergence studies similar to that shown for the UFAST SBLI case in Appendix C.

3.4. Holden's compression corner simulations

Experiments were performed in the CUBRC shock tunnel facility for a range of ramp angles varying between 27° and 36° with a freestream Mach number of about 8 [73]. The length of the plate before the corner is 995.68 mm and the ramp length is 304.8 mm (see Fig. 2(b)). Working fluid is dry air and Table 1 gives the freestream conditions for these cases. The flow is characterized by a fully developed turbulent boundary layer ahead of the interaction region. Accuracy in surface pressure measurements was estimated as $\pm 3\%$ and the experimental data for comparison is taken from [19].

The computational domain is shown with dashed lines in Fig. 2(b). The flat plate length before the corner is kept to be 0.2 m for the SBLI simulations. Separate flat plate simulations are performed, and mean and turbulence profiles are extracted at 0.8 m from the plate leading edge. These profiles then serve as inlet conditions for the ramp computations. The highest temperature encountered in these simulations is around 750 K; calorically perfect gas assumption is used in the governing equations. A no-slip isothermal-wall boundary condition is employed at the solid walls. An extrapolation condition for the mean and turbulence variables is used at the top and exit boundaries.

The SBLI simulations for these cases are performed on a body-fitted structured grid with 400×250 cells. The grid uses exponential stretching in the wall-normal direction and the height of first cell next to the wall is 1×10^{-6} m which is equivalent to about 0.5 wall units or lower. The grid is finer near the corner with a minimum cell spacing of 1×10^{-4} m and an exponential stretching is used in the upstream and downstream directions along the wall. A previous study [74] based on different $k-\omega$ models uses identical computational grid in these SBLI cases. The SST modification to eddy viscosity is not expected to alter the grid requirement significantly.

4. Results

We next apply the standard SST model, the SUQ-SST model, and EARSM of Rung et al. [49] to the test cases delineated above. This is to assess the performance of the SUQ-SST model in predicting shock/boundary layer interactions at different Mach numbers, for varying strength of interaction. Comparison is made with available experimental data, and with the predictions of existing models. A zero-pressure-gradient boundary layer is also presented as verification of the SUQ-SST model performance in the absence of shock waves.

At this point, an important distinction between the three turbulence models has to be made which results in the observed trends for the Reynolds stress, surface pressure, and skin friction distributions. Menter SST model relies on a tensorially linear relation between Reynolds stress and the strain rate tensors. However, the coefficient of this relationship, i.e. eddy viscosity μ_T , depends on the vorticity invariant, and the model allows for the transport of principal Reynolds stress. On the other hand, EARSM and SUQ-SST both use identical quadratic relation between the Reynolds stress and the strain rate tensors. The model formulation for SUQ-SST takes into account the transport of

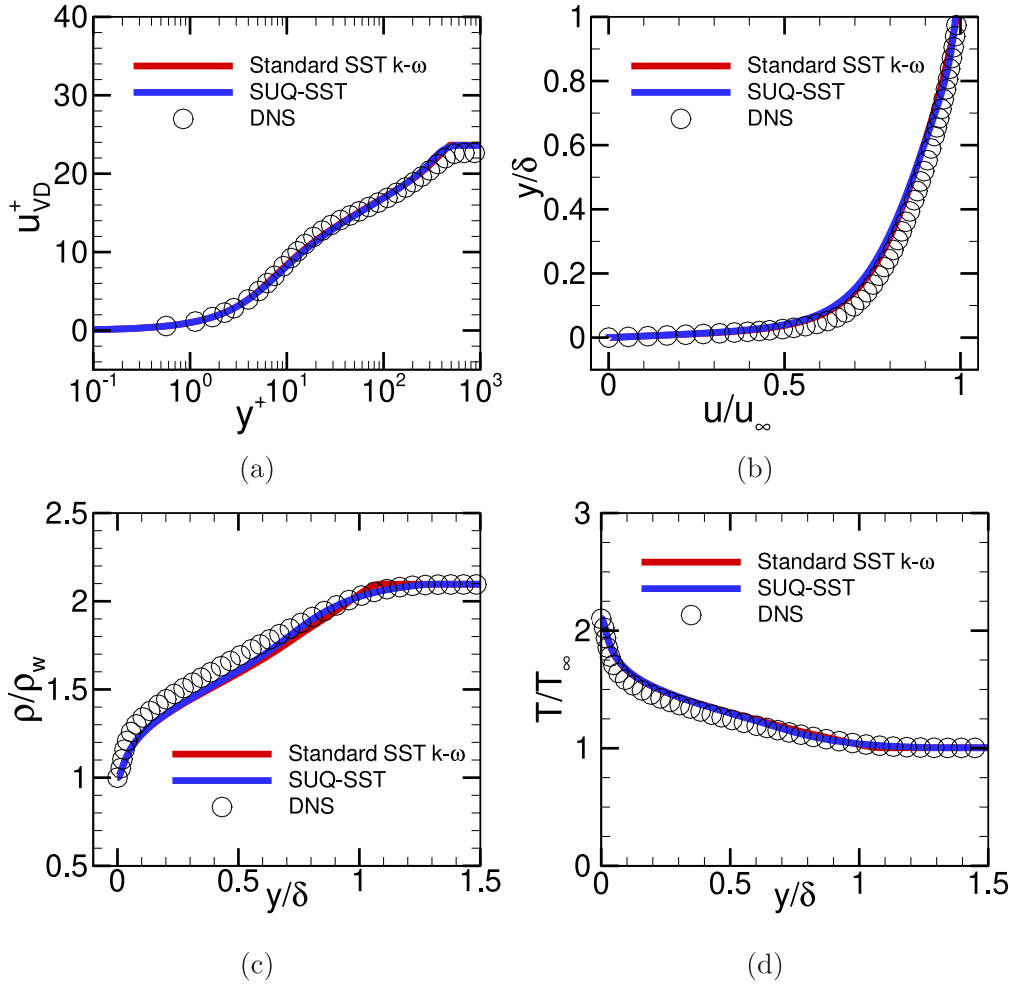


Fig. 3. Comparison of (a) Van Driest transformed mean velocity, (b) streamwise mean velocity, (c) mean density, and (d) mean temperature obtained using the standard SST and SUQ-SST models with the DNS data.

principal Reynolds stress along with an additional shock/turbulence interaction physics, which is not considered in the EARSIM. The use of different constitutive models for the Reynolds stress tensor together with the different approaches of sensitizing the eddy viscosity to mean velocity gradients results in different distribution of Reynolds stresses in the flow-field for the three turbulence models. Specifically, the three models are expected to show different behaviours in predicting the mean and turbulent quantities in the vicinity of the separation shock.

In comparison to the standard SST model, the SUQ-SST model requires additional computations of the quadratic part of the Reynolds stress components given in Eqs. (12) and (13), along with the EARSIM model parameters given in Eqs. (15) and (16). Further, the effective structure parameter is calculated using Eq. (19) and the shock function f_s is obtained using Eq. (27). These additional computations do not alter the computational requirement significantly. They need only about 6 - 7 % extra computational time per time-step than the standard SST model on identical grids. Moreover, the robustness of the standard SST model is not compromised by the modifications proposed in this work. The Courant–Friedrichs–Lewy (CFL) number used in SUQ-SST computations for each case are comparable to those in the case of SST and EARSIM models. As a result, the number of time steps and the overall computational time required to reach steady state solution by the different models are within 10% of each other.

4.1. Zero-pressure-gradient flat plate

The SUQ-SST model gives almost identical values of Reynolds shear stress as the standard SST model in a zero-pressure-gradient turbulent boundary layer. This is because the shear component of the quadratic part of Reynolds stress given in Eq. (13) has a negligible contribution in an equilibrium boundary layer. Moreover, the modified structure parameter calculated using Eq. (19) assumes values closer to 0.31 for most parts of the undisturbed boundary layer, thereby returning μ_T values similar to the standard SST model. Since, Reynolds shear stress principally dictates the growth and characteristics of a turbulent boundary layer, SUQ-SST model matches the standard SST model in resolving a turbulent boundary layer.

Fig. 3(a) plots the comparison between DNS and the two SST models for the Van Driest transformed mean velocity defined as [75],

$$u_{VD}^+ = \frac{1}{u_\tau} \int_0^{\tilde{u}} \left(\frac{\tilde{\rho}}{\rho_w} \right)^{(1/2)} d\tilde{u} \quad (28)$$

where $u_\tau = \sqrt{\tau_w/\rho_w}$ is the friction velocity, with τ_w being the wall shear stress and ρ_w is the density at the wall. The comparisons are shown at a location with Re_θ of about 2810. The SUQ-SST model gives almost identical result with the standard SST model and both match the DNS data well. This is also the case for the mean velocity, density, and temperature profiles in Fig. 3(b) – (d). This shows that modifications in terms of adding EARSIM features to the SST framework and calculating structure parameter using the new relation do not alter the boundary layer prediction capability of the SST model.

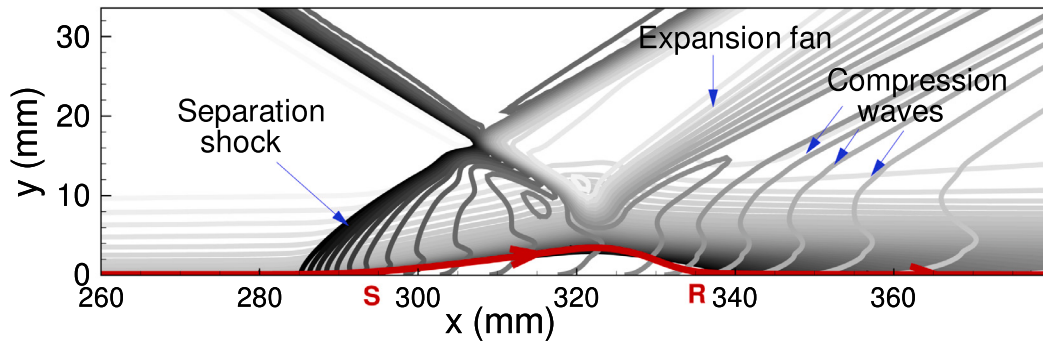


Fig. 4. Contours of Mach number superposed upon normalized pressure to show the flow structure in the UFAST SBLI case.

4.2. UFAST Mach 2.3 impinging shock case

The composite Fig. 4 shows the flow structure in an oblique shock impingement SBLI. Here, Mach number contours are used to show the boundary layer, whereas normalized pressure is used to depict the inviscid flow structures such as shock waves and expansion fan. The impinging shock in this case is strong enough to cause separation of the turbulent boundary layer. The limiting streamline identifies the separation **S** and reattachment **R** locations in Fig. 4. The separation bubble acts as a ramp and forms a separation shock. The flow turning over the separation bubble expands and eventually reattaches on the plate, forming compression waves that coalesce to form the reattachment shock.

4.2.1. Mean velocity and Reynolds stresses

Fig. 5(a) shows the comparison of the computed and experimental velocity profiles at different streamwise positions. At the inflow plane for the SBLI simulations ($x = 260$ mm), all the turbulence models correctly capture the boundary layer and give excellent match with the experiment. The SUQ-SST model predicts the interaction location slightly upstream of the experimental data and hence shows more strongly decelerated boundary layer than the experiment at separation point ($x = 300$ mm). At this location, EARSM results are closer to the experimental data. Standard SST slightly under-predicts, and it is almost identical to the SSG/LRR- ω Reynolds stress model (RSM) predictions, taken from [16]. Inside the separation region ($x = 323$ mm) and further downstream ($x = 340$ mm), the SUQ-SST model matches well with the experimental data, and is comparable to the other models.

Fig. 5(b) and (c) plot the square root of the normal Reynolds stress components in the axial direction ($-\tau_{xx}/\bar{\rho}$)^{1/2} and in the wall-normal direction ($-\tau_{yy}/\bar{\rho}$)^{1/2}. For the SST model, Boussinesq hypothesis, which linearly relates the Reynolds stresses with strain-rate, is used to compute the Reynolds stresses whereas a quadratic relationship between Reynolds stresses and strain-rate and vorticity tensors is used for the EARSM and SUQ-SST models. For RSM, Reynolds stresses are directly obtained by solving the individual Reynolds stress differential equations, as described in [16].

The SST model predicts similar levels of the normal Reynolds stress components at the corresponding axial locations. However, experimental data clearly shows higher levels of the streamwise Reynolds stress than the vertical component at each location. This tendency to predict isotropic normal Reynolds stresses is the inherent limitation of the Boussinesq hypothesis based SST model. Moreover, SST model significantly underpredicts the streamwise Reynolds stress component and overpredicts the vertical component at all the four axial locations compared to the experimental data.

The improved modelling of the Reynolds stresses in EARSM, SUQ-SST, and RSM enables them to capture the Reynolds stress anisotropy. All three models predict higher levels of streamwise Reynolds stress component than the vertical component, which is consistent with the

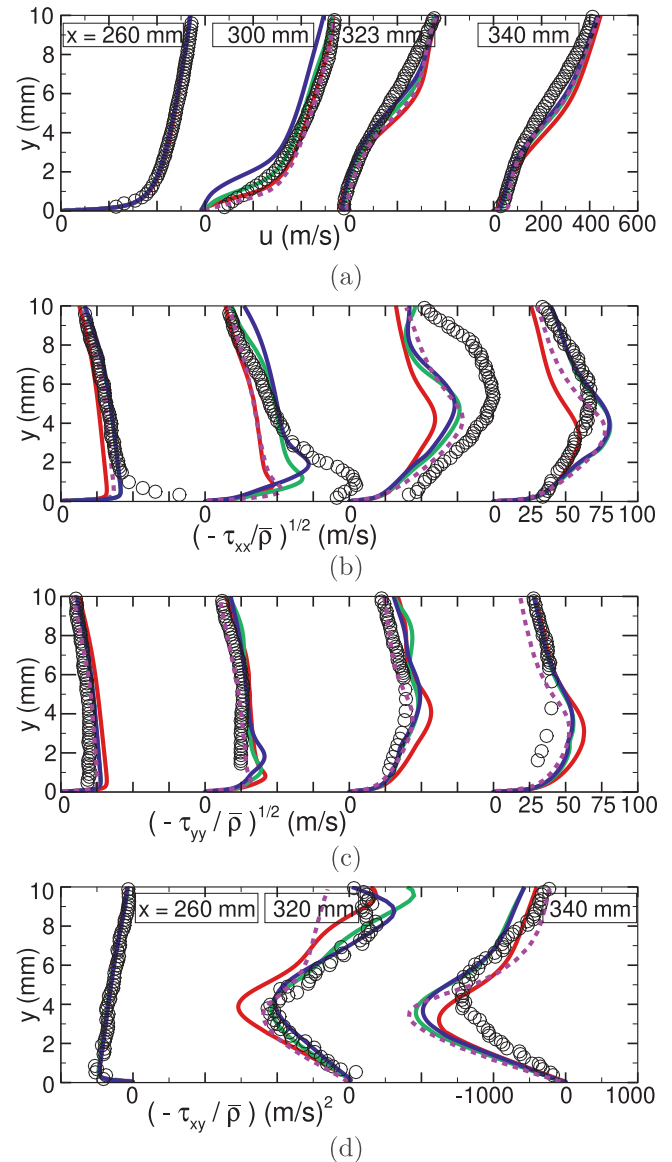


Fig. 5. Comparison of the streamwise velocity, the streamwise, transverse, and shear components of Reynolds stresses at different streamwise positions: — Standard SST ; - - - SSG/LRR- ω [16]; — EARSM; — SUQ-SST; ○ Experiment.

experiment. The SUQ-SST model significantly improves the normal Reynolds stress predictions compared to the standard SST model and brings the results closer to the experiment, although discrepancies still

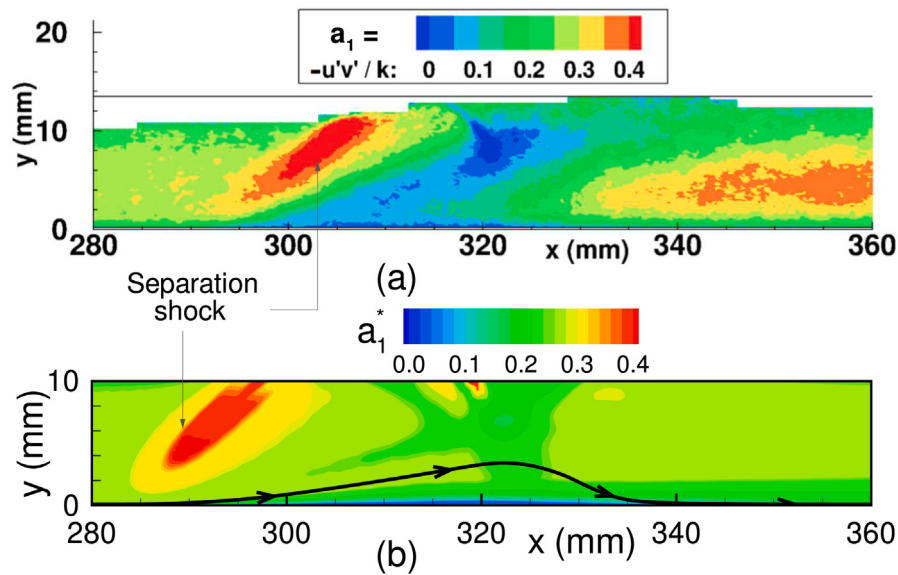


Fig. 6. Distribution of the structure parameter in the vicinity of shock/boundary layer interaction region : (a) Experimental data [31] and (b) SUQ-SST simulation.

remain. The cause of these discrepancies can be attributed to the low-frequency shock oscillations, which are observed in the experiments and not accounted for in the RANS simulations.

Fig. 5(d) shows the Reynolds shear stress comparisons within the inflow boundary layer ($x = 260$ mm) as well as in the separated flow ($x = 320$ mm) and post reattachment ($x = 340$ mm). At $x = 320$ mm, negative values of τ_{xy} in the shear layer are well reproduced by SUQ-SST, EARSM, and RSM, but significantly overestimated by the SST model. At the end of the interaction zone ($x = 340$ mm), SST gives reasonable results, whereas the other models overpredict the maximum shear stress level. On the other hand, the shear-layer thickness is underpredicted by SST and RSM, whereas SUQ-SST and EARSM predict a closer match with the experiment. Overall, predictions of the SUQ-SST model are found to be comparable to the EARSM for this test case, and they are a significant improvement over the standard SST model results.

4.2.2. Structure parameter distribution

Fig. 6(a) shows the behaviour of the structure parameter, calculated as the ratio between Reynolds shear stress and turbulence kinetic energy, in the interaction region using the UFAST experimental data [31]. The structure parameter increases to a peak value of 0.4 in the separation shock region from its equilibrium value of about 0.3 in the incoming boundary layer. It then decreases post the separation shock in the first half of the separation region ($x < 320$ mm) reaching levels below 0.1 and then again increases in the latter half ($340 > x > 320$ mm). Near the centre of interaction at around $x = 320$ mm, flow expansion over the separation bubble decreases the structure parameter significantly. It again increases post reattachment ($x > 340$ mm), possibly in the reattachment compression waves, well beyond the values characteristic of equilibrium boundary layers.

Fig. 6(b) shows the effective structure parameter distribution for the SUQ-SST model as calculated using Eq. (19). The model prediction in the SBLI region is qualitatively similar to the experimental data. Specifically, the model predicts an increase in the structure parameter values at the separation shock with levels almost identical to the experimental data. The model also correctly predicts a decrease in structure parameter values in the first part of the separation ($x < 320$ mm) and gives levels comparable to the experimental data in the latter part ($340 > x > 320$ mm). The model shows a slight increase in the structure parameter values from 0.25 ($x \sim 320$ mm) to 0.31 ($x > 340$ mm) in the recovering boundary layer, however the growth is not to the extent as observed in the experiments. Meanwhile, the levels near

the centre of interaction at around $x = 320$ mm due to the expansion effect are higher to that observed in the experiment. Note that, we have retained the forms of the model parameters β'_2 , β'_3 , and C_μ^* from the original EARSM to construct the structure parameter in Eq. (19), but they can be modified to improve the predictions. However, the objective here is to get correct values in the vicinity of the separation shock ($x < 320$ mm) since this region majorly decides the separation. The values of the structure parameter post the separation shock region ($x > 320$ mm) including the recovery region are not expected to affect the predictions for separation shock location and the separation bubble size significantly.

4.3. Mach 5 impinging shock cases

The different turbulence models are next applied to oblique shock impingement SBLI at Mach 5, and the results are compared in terms of the structure parameter in Fig. 7. The limiting streamline, which originates from the separation point and separates the recirculating flow inside the separation bubble from the flow turning over it, is used to identify the separation S and reattachment R points in each case. Results are presented for the strongest interaction of 14° deflection angle; other cases show a similar trend.

SST model uses a constant value of the structure parameter $a_1 = 0.31$ inside the boundary layer (Fig. 7(a)) including regions of shock-induced adverse pressure gradients. This is in conflict with the experimental observations as discussed earlier. To represent the correct SBLI physics, EARSM features are introduced in the SST framework and the structure parameter is made a function of the local strain rate, as per Eq. (19). This modification is then able to capture the changes in structure parameter in the separation and reattachment shocks inside the boundary layer (Fig. 7(b)), where a peak value of about 0.48 is seen. By comparison, SUQ-SST correctly captures the changes in structure parameter in the entire shock structure (Fig. 7(c)), including regions outside the boundary layer. This includes the separation shock, the reattachment shock, and the other shocks shown in the figure. The structure parameter reaches a maximum value of 0.4 in the shock regions, as prescribed by the SU limiter.

The variation of the structure parameter has a direct bearing on the flow separation predicted by the model. The EARSM structure parameter given by Eq. (19) moves the separation point slightly upstream of the standard SST model. The SU-limiter moves it further upstream to give a larger separation bubble. The three models give similar locations of reattachment point R as shown in Fig. 7.

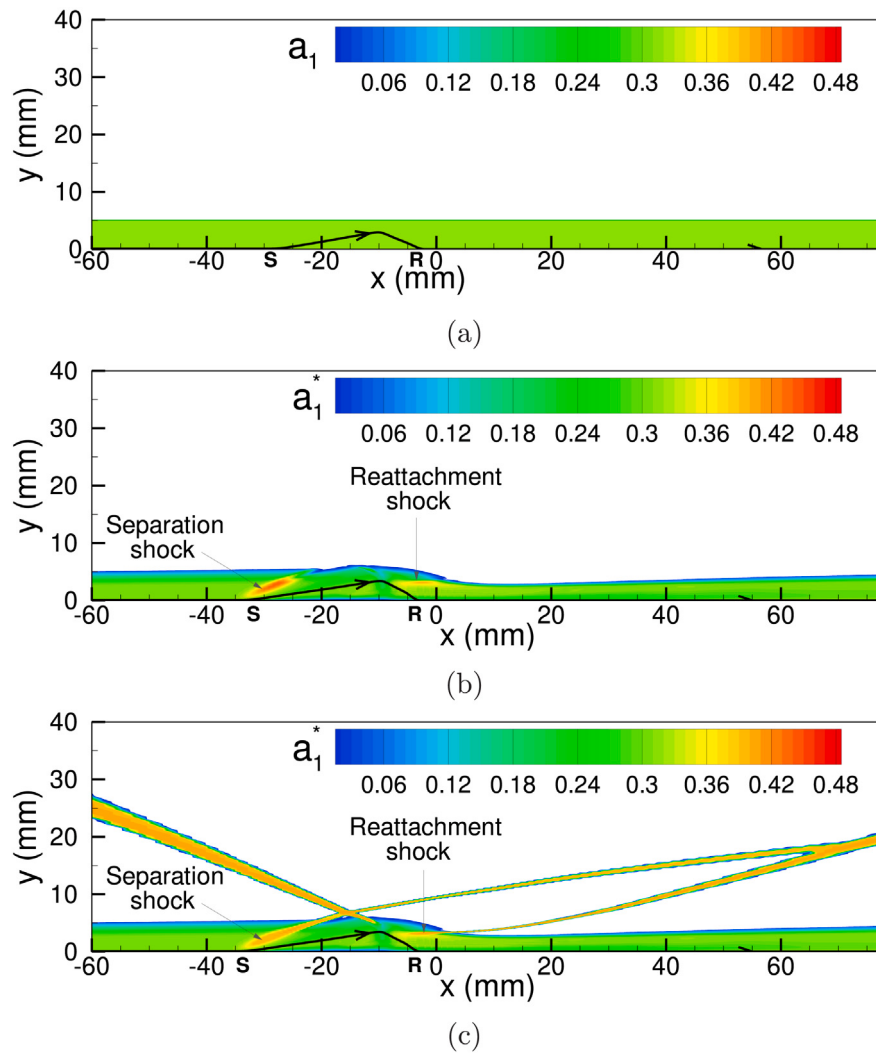


Fig. 7. Distribution of the structure parameter in the Schulein's 14° SBLI flowfield obtained using (a) standard SST $k-\omega$ model and SUQ-SST model (b) Eq. (19) and (c) Eq. (23).

Fig. 8 compares the wall pressure and skin friction coefficient distributions in the SBLI region obtained using the RANS models with the experimental data. The standard SST $k-\omega$ model predicts the separation shock location downstream of the experiment and significantly under-predicts the separation bubble size for the 10° and 14° cases. EARSIM gives similar under-prediction for 10°, but shows improvement over the SST model for the 14° case. However, discrepancies still remain.

The SUQ-SST model improves the results significantly compared to the standard SST model. For the 10° case, the separation location is predicted accurately, both in terms of the surface pressure rise and the zero crossing of the skin friction coefficient. A distinct pressure plateau is observed, unlike the experimental data. The skin-friction and surface pressure post-reattachment for the SUQ-SST model also agree well with the experiment. The strongest interaction, with 14° shock-generator angle, shows similar trends and SUQ-SST predictions are better compared to the standard SST model. The separation bubble size obtained using SUQ-SST is within 9% of the experiment, as opposed to 27% for SST.

For the weakest interaction case with 6° shock generator angle, all the turbulence models predict near incipient separation and agree well with experiment. There are no major differences between SST, EARSIM, and SUQ-SST for the wall pressure and skin friction profiles for this case. Overall, SUQ-SST consistently gives good predictions for varying strengths of interactions and match the experiment better compared to SST and EARSIM.

The model trends for the separation size predictions can be explained in terms of the Reynolds stress contributions to the x - and y -momentum equations given by the underlined terms in Eq. (2). Fig. 9 shows one such comparison between the different turbulence models for the Schulein's 14° case. Contributions of the fluxes of the Reynolds stresses to the x -momentum are found to be significantly higher compared to their contribution to the y -momentum in the vicinity of the separation shock including the separated shear layer for all the models. The overall contribution of Reynolds stresses to the x -momentum in the separation shock foot and in the separated shear layer mainly determines the separation location and the separation length; higher the contribution smaller is the bubble length.

The three models predict comparable Reynolds stress contribution near the separation shock foot. However, the SST model predicts significantly larger values in the separated shear layer resulting in a higher overall contribution compared to the EARSIM and SUQ-SST model. This results in SST predicting the smallest separation bubble amongst the three models. Meanwhile, the EARSIM and SUQ-SST model give comparable overall Reynolds stress contribution in the separation shock foot and the separated shear layer, and hence predict almost similar separation bubble sizes. For the 10° case, SST and EARSIM are found to give similar results for the Reynolds stress contributions to the x -momentum, while SUQ-SST model gives a lower contribution than these models in the vicinity of the separation shock. This results in a bigger separation bubble for the SUQ-SST model compared to the

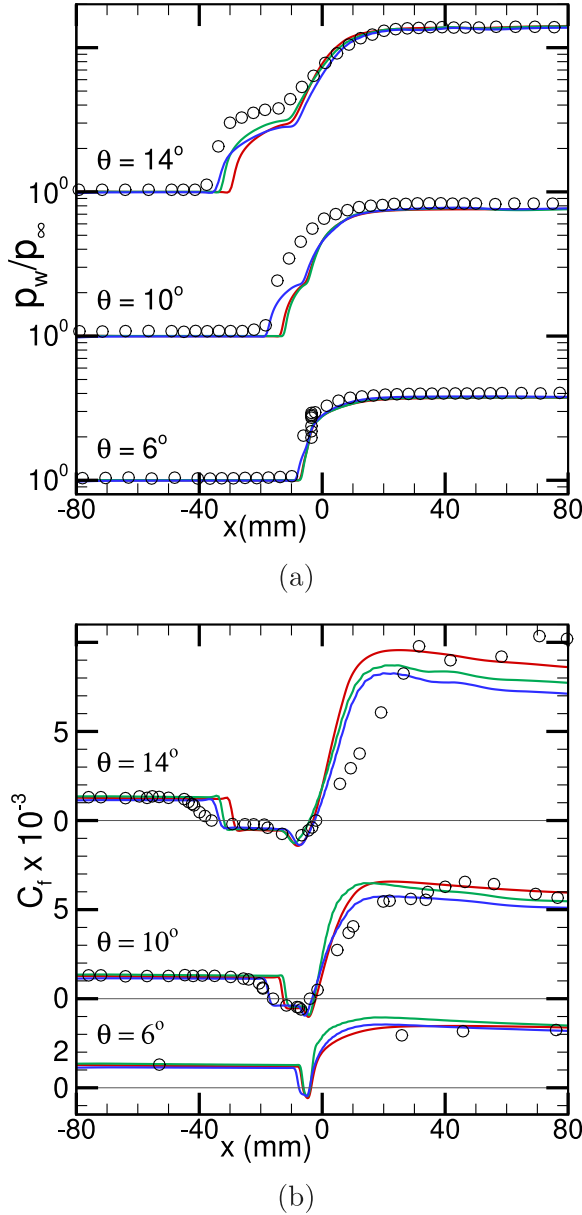


Fig. 8. Comparison of (a) wall pressure and (b) skin friction coefficient obtained using different $k-\omega$ models with the experimental data for varying shock generator angles : Standard SST $k-\omega$; EARSIM; SUQ-SST; $\circ$ Experiment.

EARSIM and SST model, which predict similar separation. On the other hand, the three models predict comparable Reynolds stress contributions to the momentum equations in the vicinity of the separation shock for the 6° case, which result in similar surface pressure and skin friction distributions.

4.4. Mach 8 compression corner cases

For an incoming flow with Mach number of about 8, the range of ramp angles considered in the experiments resulted in near incipient to fully separated flows. The incoming boundary layer is almost identical in these cases, so the effect of shock strength on the interaction length and flow separation can be studied. The strength of the shock waves also affects the distribution of the structure parameter as shown in Fig. 10. The SUQ-SST model captures the effect of shock strength on separation quite well and predicts near incipient separation for 27°

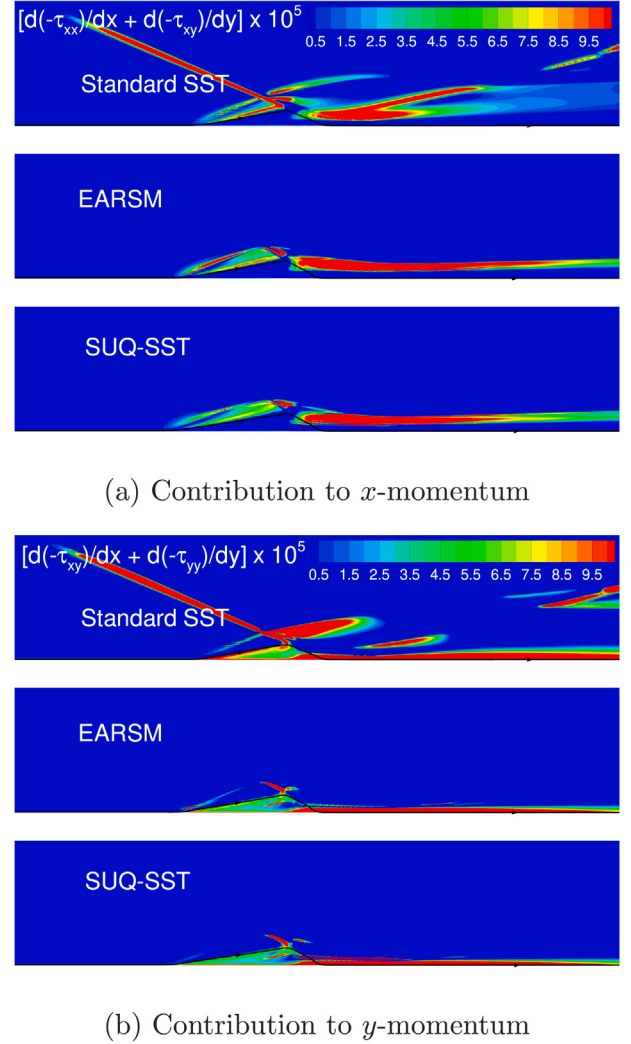


Fig. 9. Contribution of the Reynolds stresses to the x- and y-momentum equations obtained using the different models for the 14° Schulein's impinging SBLI case.

case, mild separation for 30° case, and large separations for 33° and 36° cases, which is consistent with the experimental observations. The separation and reattachment locations are marked with S and R, respectively in Fig. 10.

The structure parameter increases from its equilibrium value of 0.31 and reaches a peak value of 0.4 in the separation and reattachment shocks inside the boundary layer and the ramp-induced shock in the inviscid region (Fig. 10). The shock-shock interaction over the ramp in the separated cases results in Edney type VI interference that emanates an expansion fan. Expansion effect reduces the values of structure parameter below 0.31 in the reattached boundary layer and the magnitude decreases with an increase in the expansion effect seen in terms of increasing ramp angle. The structure parameter eventually recovers back to its original value.

Fig. 11 shows the wall pressure distribution for varying ramp angles as obtained using the different RANS models and experiments. For the weakest interaction i.e. 27° case, all the turbulence models give similar results and predict incipient separation matching the experimental data. Experiments show mild separation for the 30° case and SUQ-SST predicts the pressure variation accurately with SST giving similar results. EARSIM gives slightly delayed separation and predicts separation shock location downstream of the experiment for this case.

The 33° and 36° cases give fully separated flows and SUQ-SST predicts the separation shock location in close agreement with the

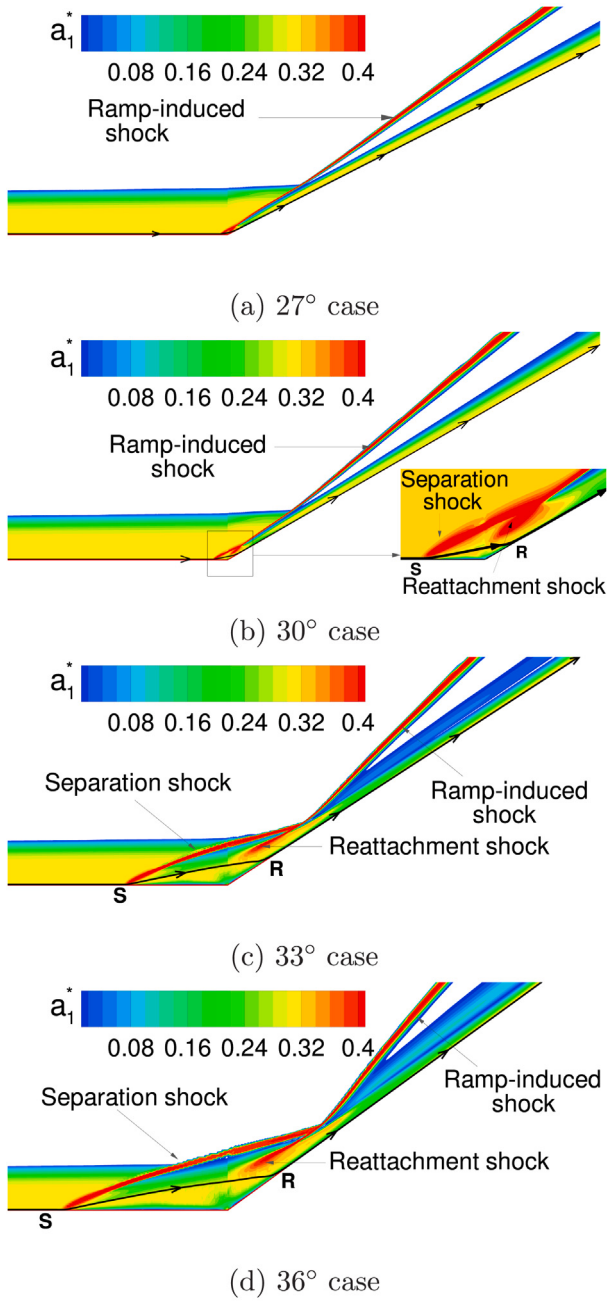


Fig. 10. Distribution of the effective structure parameter a_1^* in the Mach 8 SBLI flowfield for different ramp angles.

experiment for both the cases. The model also predicts the pressure distribution along the ramp and the peak reattachment pressure in close agreement with the experimental data. On the other hand, SST and EARSF give strikingly inconsistent results. Both models underpredict the separation regions considerably for the 33° case, while giving significant overprediction for the 36° interaction. The two models also underpredict the peak pressure for the 33° case, while giving the peak location much downstream of the experiment for the 36° case.

Similar to the Schulein's cases, the surface pressure and skin friction distribution results obtained using the three models can be explained in terms of the overall Reynolds stress contributions to the x - and y -momentum equations given by the underlined terms in Eq. (2). The three models predict similar Reynolds stress contributions to the momentum equations for the 27° and 30° cases resulting in similar

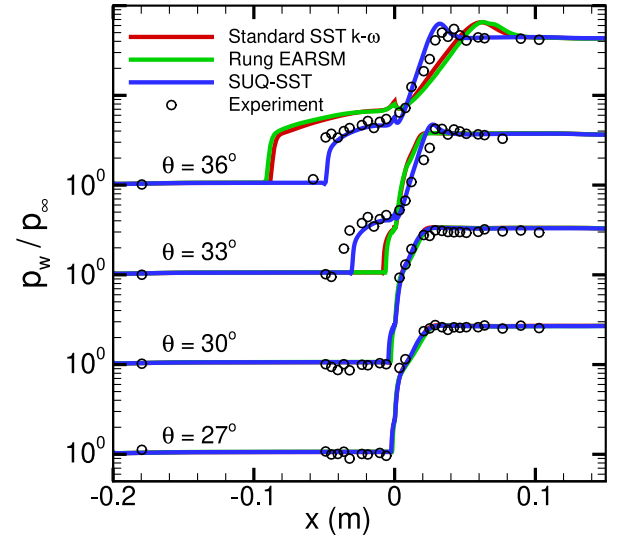
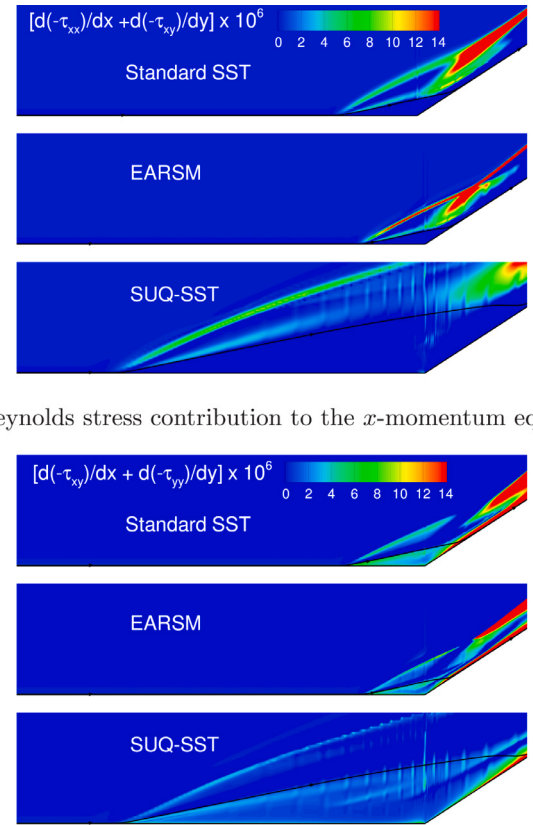


Fig. 11. Comparison of wall pressure obtained using different k - ω models with the experimental data for the Mach 8 SBLI cases at varying ramp angles.

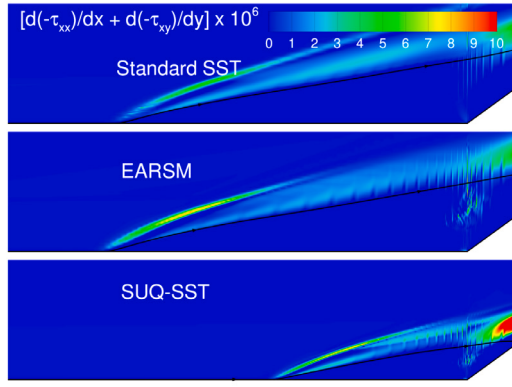
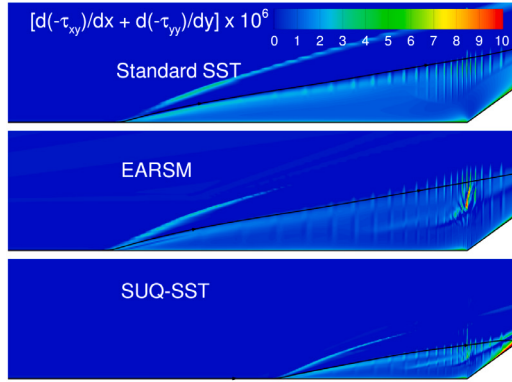


(a) Reynolds stress contribution to the x -momentum equation

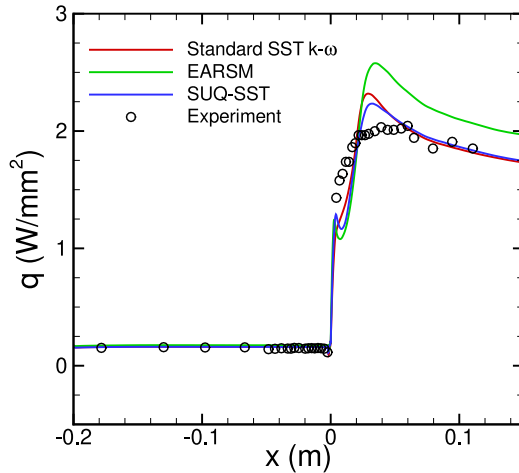
(b) Reynolds stress contribution to the y -momentum equation

Fig. 12. Comparison between the different model predictions for the Reynolds stress contribution for the Holden's 33° case.

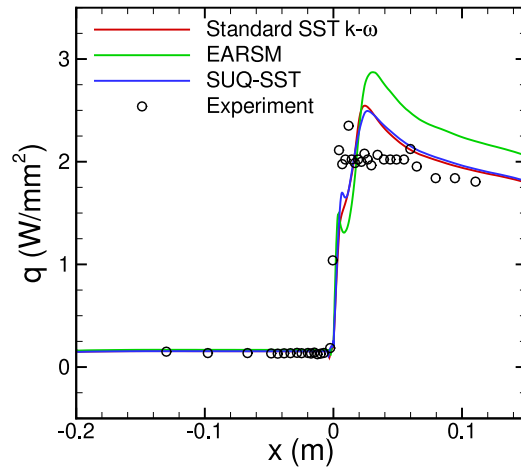
separations for these cases. Fig. 12 shows the Reynolds stress contributions to the momentum equations for the 33° case. The three models predict comparable Reynolds stress contribution to the x -momentum in the separation shock foot. Unlike the Schulein's cases, y -momentum Reynolds stress contribution becomes comparable to the x -momentum contribution in the separation shock foot for these high Mach number

(a) Reynolds stress contribution to the x -momentum equation(b) Reynolds stress contribution to the y -momentum equation**Fig. 13.** Comparison between the different model predictions for the Reynolds stress contribution for the Holden's 36° case.

cases. EARSM and SST give significantly higher values for the y -momentum contribution compared to the SUQ-SST model. Also, the Reynolds stress contributions to the x -momentum in the separated shear layer is significantly higher for the EARSM and SST model than SUQ-SST.



(a) 27° case



(b) 30° case

Fig. 14. Comparisons for wall heat transfer rate predictions between different turbulence models and experimental data for the two weak interaction cases of Holden.

The Reynolds stress contributions to the x - and y -momentum equations obtained using the three turbulence models for the 36° case are shown in Fig. 13. Similar to the 33° case, contributions of Reynolds stresses to the y -momentum in the separation shock region are comparable to the x -momentum contributions. The three models predict almost similar Reynolds stress contributions to the x - and y momentum in the separation shock foot. However, contributions to the x -momentum in the separated shear layer are higher for the SUQ-SST model compared to the other two models. This causes a larger overall Reynolds stress contribution to the momentum equations for the SUQ-SST model giving a significantly smaller separation compared to the EARSM and SST model. Similar to the 33° case, the overall Reynolds stress contributions in the vicinity of the separation shock predicted by the EARSM and SST model are comparable, and hence these models predict almost similar bubble sizes. Note that the visible breaks at regular space intervals in the levels of the Reynolds stress contributions in Figs. 12 and 13 correspond to the inter-processor boundaries and are an artifact of the post-processing of the data.

4.5. Wall heat transfer rate predictions

Surface heat transfer prediction, especially the reattachment point heat transfer, is another important quantity of engineering interest for high-speed flows with SBLIs. The distribution of wall heat transfer rates follow a similar pattern as the surface pressure; heat transfer is almost constant before the interaction, then it starts to steeply rise near the separation location reaching peak values near the reattachment point followed by a gradual drop. The trend of wall heat transfer rate distribution is similar regardless of the geometric configuration, i.e compression corner, oblique impinging/reflected shock, cone-flare, etc.

Fig. 14 shows the comparisons for wall heat transfer predictions between different turbulence models and the available experimental data of Holden et al. [73], and the results correspond to compression ramp angles of 27° and 30°. As discussed earlier, the two weak interaction cases show small separations. The three models show a steep rise in heat transfer after the separation location which matches with the experiment for both the cases. The post-reattachment ($x > 0.04$) trend for all the models is qualitatively similar. EARSM gives significant overpredictions of the peak heat transfer rates post reattachment ($x > 0$) with an error of about 25%. On the other hand, standard SST and SUQ-SST models give almost similar results and match the experimental data better than the EARSM for the two cases. The SUQ-SST model

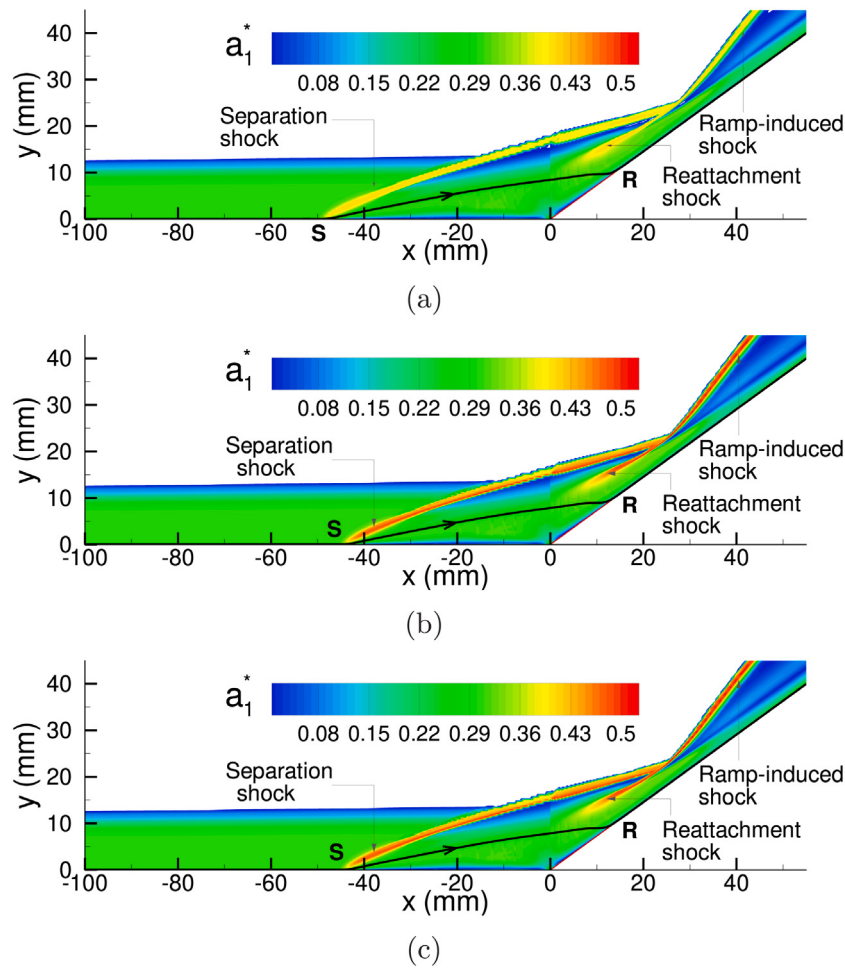


Fig. 15. Distribution of the effective structure parameter a_1^* in the Holden's 36° SBLI case obtained using SUQ-SST model with (a) SU limiter = 0.4; (b) SU limiter = 0.51; (c) SU limiter = 0.62.

predictions are within 10% of the experimentally measured peak heat flux for both the cases.

Generally, correct prediction of the surface pressure and skin friction distributions together with the separation bubble size does not guarantee the correct prediction of wall heat transfer rates for the hypersonic SBLI cases [74,76,77]. The limitation can be attributed to the use of Reynolds analogy and modelling of the turbulent heat-flux vector in terms of a turbulent Prandtl number [76,77]. This is true for the standard and modified turbulence models considered in this paper. Large discrepancies for heat transfer predictions are expected in case of strong interaction cases that show large separations. Realistic predictions for hypersonic reattachment point heat transfer can be achieved by constructing additional model equations to compute the heat-flux vector, and this is beyond the scope of this paper. Alternatively, variable turbulent Prandtl number models [74] that still use the temperature gradient approximation for modelling the turbulent heat-flux vector, are also expected to bring down the error margin in predicting the reattachment point heat transfer rates.

4.6. Sensitivity to the SU limiter

The SU limiter is obtained using Menter-like analysis of Sinha et al. [55] shock-unsteadiness model at a shock wave. The analysis gives the value of structure parameter in the shock regions in the range of 0.62 for weak shocks to 0.4 in high Mach number flows. The SU

limiter, given by a_1'' in Eq. (22), brings in the effect of shock strength and decides the maximum value of structure parameter in the shock regions. Thus, it directly affects the eddy viscosity distribution in the SBLI regions, and thus can alter the separation and interaction lengths.

Fig. 15 shows the distribution of the effective structure parameter obtained using the SUQ-SST model using different values of the SU limiter. Specifically, the nominal value $a_1'' = 0.4$, the arithmetic mean of the extreme values $a_1'' = 0.51$, and the weak-shock limiting value $a_1'' = 0.62$, are used for the strongest interaction in the 36° Mach 8 case. All three cases presented in Fig. 15 show identical shock topology, where the structure parameter increases from its starting value of 0.31 in the boundary layer to the limiting value in the separation and reattachment shocks. A higher limiting value in the $a_1'' = 0.51$ solution moves the separation location S downstream, compared to the nominal $a_1'' = 0.4$ case. This results in a slight decrease in interaction length as seen from the wall pressure distribution (Fig. 16). Further increasing the SU limiter to 0.62 shows negligible change in the structure parameter distribution, location of the separation shock, and the surface pressure distribution.

The sensitivity of the results to the value of the SU limiter is found to be highest for the 36° case presented above. This is the strongest interaction among the cases considered in this work. The strain rates are therefore expected to be significantly higher than the other interactions, either at a lower Mach number or a lower deflection angle. For example, the SUQ-SST model predictions show almost no

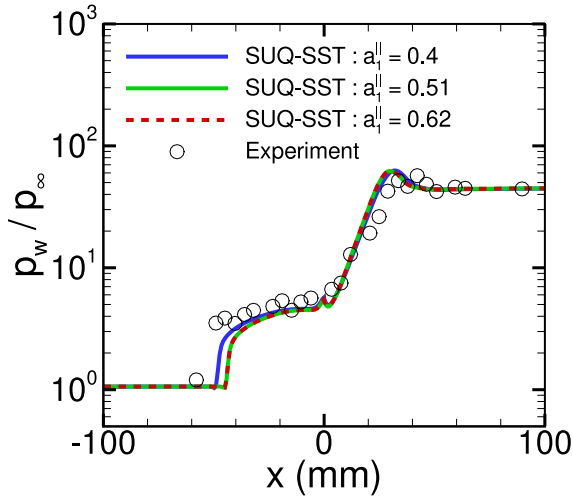


Fig. 16. Sensitivity of wall pressure distribution to the SU limiter values for the SUQ-SST model for Holden 36° compression corner SBLI case.

sensitivity to the SU limiter value for the 14° case at Mach 5, where the shock strength is about three times weaker than the 36° Mach 8 interaction.

5. Conclusion

The paper presents an advanced form of SST k - ω model for applications involving high-speed flows with shock/turbulent boundary layer interaction. The new model, called as SUQ-SST, is developed by introducing features of EARSIM in the SST framework. A quadratic relation between the Reynolds stresses and strain and vorticity tensors by Rung et al. [49] is used for this purpose. The structure parameter is modified and it depends on the local strain rate and the anisotropy parameter. The modified structure parameter increases in response to a shock-induced adverse pressure gradient, as observed in SBLI experiments. However, at a shock wave, strain rates can become significantly high that may lead to unphysical values of the structure parameter. To prevent this, the value of the structure parameter at a shock wave is limited using the shock-unsteadiness (SU) limiter. The limiting value is obtained from a Menter-like analysis of the shock-unsteadiness model of Sinha et al. [55] at a shock wave. The SU limiter is similar, but more formal and based on physical analysis, than the use of ad-hoc eddy-viscosity limiters found in the literature.

The new SUQ-SST model retains the capabilities of the standard SST model in resolving a zero-pressure-gradient turbulent boundary layer. The model is tested against a number of two-dimensional canonical SBLI flows, including UFAST Mach 2.3 experiment, Mach 5 impinging shock cases, and Mach 8 compression corner flows. The selected test cases cover a wide range of flow regime from supersonic to hypersonic speeds, and near incipient to fully separated flows. The SUQ-SST model is able to accurately capture the behaviour of the structure parameter in the SBLI regions in accordance with the UFAST experimental data. The added physics leads to significantly improved predictions of Reynolds stresses and their anisotropy compared to the standard SST model. The SUQ-SST model is also able to consistently predict the separation shock location and separation bubble size. It thus clearly outperforms the standard SST model in predicting a wide range of SBLI flows.

CCRediT authorship contribution statement

Pratikkumar Raje: Conceptualization, Methodology, Software, Validation, Formal analysis, Investigation, Data curation, Writing - original

draft, Visualization. **Krishnendu Sinha:** Writing - review & editing, Supervision, Data curation, Resources, Project administration, Funding acquisition.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Appendix A. Menter SST k - ω model parameters

The blending functions for the Menter SST k - ω model are given by,

$$\begin{aligned}
 F_1 &= \tanh(\arg_1^4) \\
 \arg_1 &= \min \left[\max \left(\frac{\sqrt{k}}{\beta^* \omega d}, \frac{500\nu}{d^2 \omega} \right); \frac{4\rho\sigma_{\omega_2} k}{C_{D_{k\omega}} d^2} \right] \\
 C_{D_{k\omega}} &= \max \left(2\rho\sigma_{\omega_2} \frac{1}{\omega} \frac{\partial k}{\partial x_j} \frac{\partial \omega}{\partial x_j}, 10^{-20} \right) \\
 F_2 &= \tanh(\arg_2^2) \\
 \arg_2 &= \max \left(2 \frac{\sqrt{k}}{\beta^* \omega d}, \frac{500\nu}{d^2 \omega} \right)
 \end{aligned} \tag{A.1}$$

and d is the shortest distance from the wall. Each of the constants in Eqs. (8) and (9) is a blend of an inner (1) and outer (2) constants, blended via:

$$\phi = F_1 \phi_1 + (1 - F_1) \phi_2 \tag{A.2}$$

where ϕ_1 represents constant 1 and ϕ_2 represents constant 2. The constants are :

$$\begin{aligned}
 \gamma_1 &= \frac{\beta_1}{\beta^*} - \frac{\sigma_{\omega_1} \kappa^2}{\sqrt{\beta^*}}, \gamma_2 = \frac{\beta_2}{\beta^*} - \frac{\sigma_{\omega_2} \kappa^2}{\sqrt{\beta^*}} \\
 \sigma_{k_1} &= 0.85, \sigma_{\omega_1} = 0.5, \beta_1 = 0.075 \\
 \sigma_{k_2} &= 1.0, \sigma_{\omega_2} = 0.856, \beta_2 = 0.0828 \\
 \beta^* &= 0.09, \kappa = 0.41
 \end{aligned} \tag{A.3}$$

Appendix B. Log layer analysis of Rung et al. [49] EARSIM

Log layer is defined as the portion of the boundary layer sufficiently distant from the surface that molecular viscosity is negligible relative to eddy viscosity, yet close enough for convective effects to be negligible [36]. Neglecting the convection and pressure gradient (zero-pressure-gradient) and the molecular viscosity relative to the eddy viscosity, we have from the x -momentum equation,

$$\frac{\partial}{\partial y} \left[\mu_T \frac{\partial \tilde{u}}{\partial y} \right] = 0 \tag{B.1}$$

Integrating across the log layer thickness, we get

$$\mu_T \frac{\partial \tilde{u}}{\partial y} = \tau_{xy} = \text{constant} \tag{B.2}$$

As Bradshaw's relation hold good in a log layer and noting that Reynolds shear stress is the principal contributor to TKE production [36], we have

$$\tau_{xy} = \bar{\rho} a_1 k \tag{B.3}$$

Equating Eqs. (B.2) and (B.3), we get

$$\frac{\partial \tilde{u}}{\partial y} = \frac{\bar{\rho} a_1 k}{\mu_T} \tag{B.4}$$

Now, for Rung EARSIM, eddy viscosity is given by Eq. (14) and substituting it in Eq. (B.4) we get,

$$\frac{\partial \tilde{u}}{\partial y} = \frac{C_\mu}{C_\mu^*} a_1 \omega \tag{B.5}$$

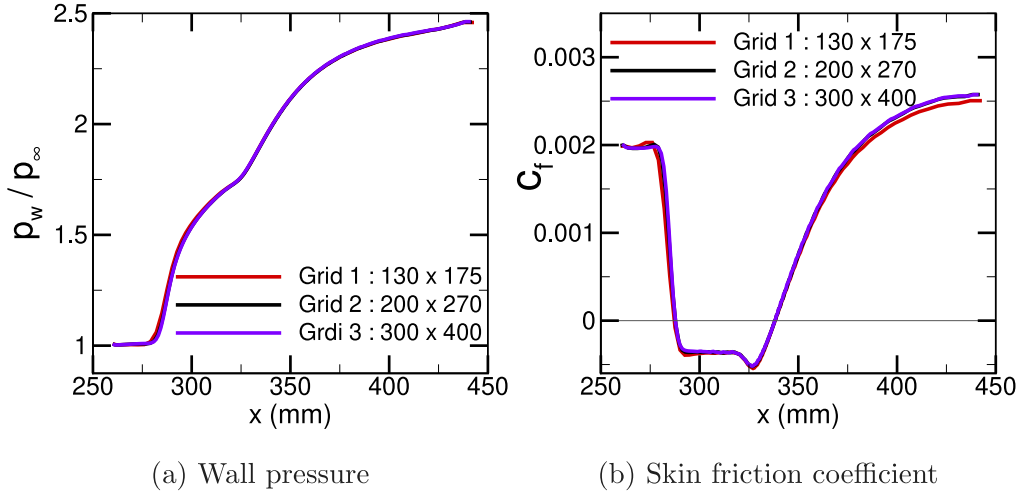


Fig. C.1. Results obtained on three different grids using SUQ-SST model for the UFAST SBLI case.

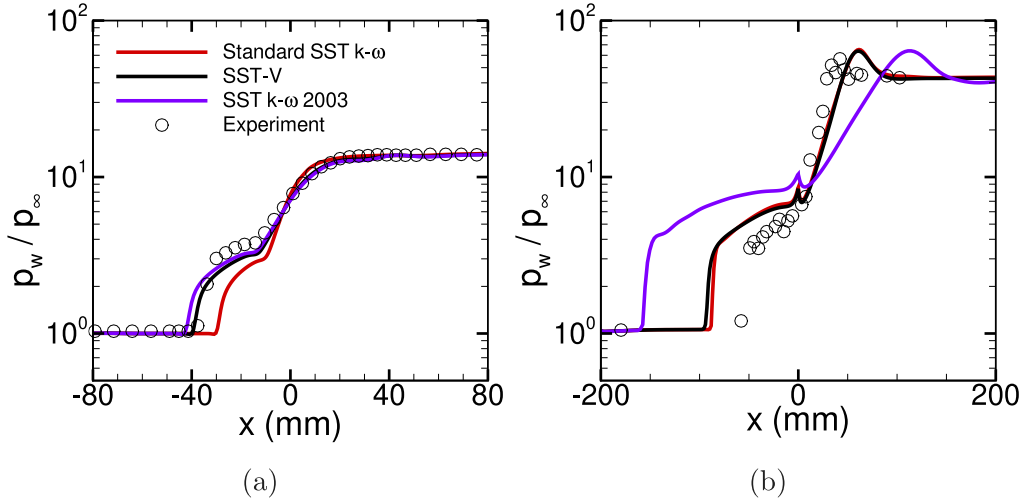


Fig. D.1. Comparison of wall pressure obtained using different variants of SST $k-\omega$ model with the experimental data for (a) Schulein's 14° impinging shock and (b) Holden's 36° compression corner cases.

We generalize the velocity gradient in the above equation in terms of the magnitude of the deformation-rate tensor D and noting that $\tau = 1/(C_\mu \omega)$, we get

$$a_1 = C_\mu^* \tau D \quad (\text{B.6})$$

Appendix C. Grid convergence

Extensive grid convergence studies have been carried out for the SBLI test cases using SUQ-SST model by systematically varying the number of grid points in both x and y directions as well as refining the cell size in critical regions. The methodology adopted in performing the grid refinement studies is explained below for the UFAST Mach 2.3 impinging shock case. Grid convergence studies for the SBLI cases of Schulein and Holden follow a similar methodology and are not presented here for the sake of brevity.

For the UFAST case, three successively refined grids with 130×175 , 200×270 , and 300×400 finite volume cells are used for the grid convergence study. The grids are successively refined by a factor of about 1.5 in each direction. The near wall spacing and the cell size at the interaction location are both reduced by a factor of 1.5 for

each successively refined grid. Fig. C.1 shows the skin friction and wall pressure distribution for the three grids. Skin friction is found to be more sensitive to grid refinement than the wall pressure and so it is used as an indicator to arrive at a grid converged solution. While surface pressure is almost identical for the three grids, skin friction shows noticeable difference between the 130×175 grid and the other two finer grids. The two finer grid solutions almost overlap for the skin friction with a maximum variation of less than 0.1% between them. So, 200×270 grid is considered for the computations.

Appendix D. Performance of linear eddy viscosity based SST $k-\omega$ model variants

The standard SST $k-\omega$ model has a number of variants with each variant aimed at removing certain deficiencies of the original model. Here, we show the performance of two of its major variants that are widely used in practice. First one is the SST-V model [78] which uses the exact same standard SST model equations except that the production term in both k and ω model equations is replaced with vorticity-based production of the form

$$P = \mu_T \Omega^2 - \frac{2}{3} \rho k \delta_{ij} \frac{\partial u_i}{\partial x_j} \quad (\text{D.1})$$

This version is sometimes favoured for hypersonic flow applications, particularly when strong shocks are present to avoid numerical difficulties associated with the use of exact the production term. The other variant is the SST 2003 version [78] wherein eddy viscosity definition is modified and strain-rate magnitude S is used instead of the vorticity magnitude Ω in Eq. (11). Also, the production of k and ω is replaced by $\min(P, 10\beta^* \rho \omega k)$, with P being the exact production term in the k and ω equations of the standard SST model and $\beta^* = 0.09$. This form was proposed to remove the stagnation point anomaly of the standard SST model. These two variants either try to limit the k and ω production or eddy viscosity or both.

The performance of these model variants is tested using Schulein's 14° impinging shock and Holden's 36° compression ramp cases. These two cases have different length scales of interaction with Holden's case having about 3 times stronger shock strength and about 1.5 times bigger separation than the Schulein's case. As discussed earlier, the standard SST model significantly underpredicts the separation for the Schulein's case while giving considerable overprediction for the Holden's case. As the two variants either limit the production of k and ω and/or eddy viscosity, they are expected to give bigger separation than the standard SST model for flows with strong interactions. This can be seen in Fig. D.1 which plots the surface pressure distribution obtained using the different SST variants and experimental data for the Schulein's 14° and Holden's 36° SBLI cases. For the Schulein's 14° case, both variants predict significantly bigger separation than the standard SST and in fact improve the results while giving slight overpredictions. On the other hand, the predictions become worse for the Holden's 36° case, with SST-2003 version giving about 10 times bigger separation than the experiment. Thus, the different variants of SST model should be cautiously used for high-speed flow applications with shock-induced flow separation.

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