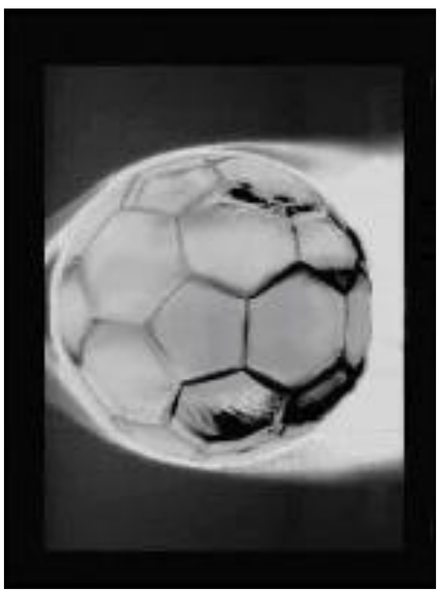
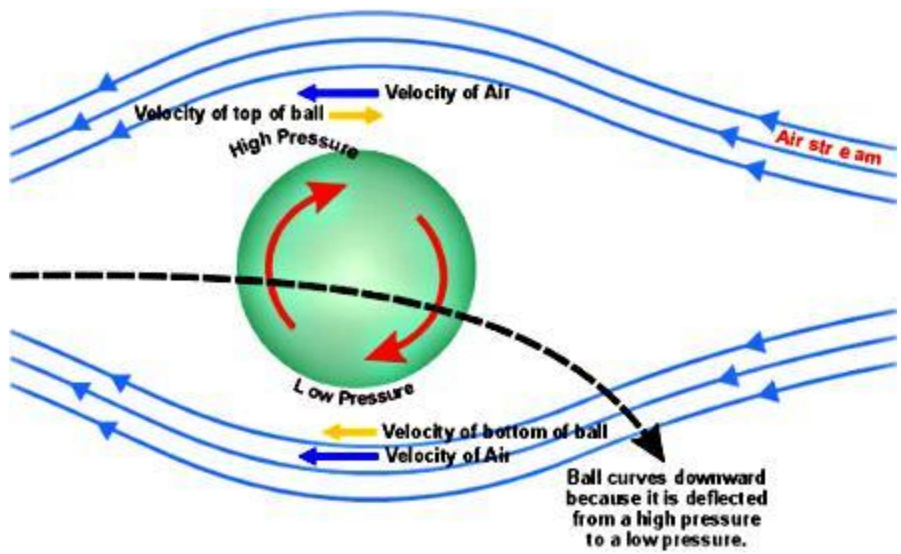
Flow past a rough cylinder with rotation

- Project under the guidance of Prof. Rama Govindarajan





Motivation

- Rotation, roughness are important in determining vortex shedding patterns
- How do they both couple and affect vortex shedding?
- How do they affect the critical Re at which vortex shedding appears?

Methodology

- Finite difference method (BTCS)
- Cylindrical coordinates
- Stream function-vorticity formulation
- Dirichlet B.C.s on boundaries
- Poisson's equation solved using FFT in theta direction + TDMA in the radial direction

Equations

$$\frac{\partial \omega}{\partial t} + \left(u, \frac{v}{r}\right) \cdot \left(\frac{\partial \omega}{\partial r}, \frac{\partial \omega}{\partial \theta}\right) = \frac{1}{\mathrm{Re}} \left(\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial \omega}{\partial r}\right) + \frac{1}{r^2}\frac{\partial^2 \omega}{\partial \theta^2}\right)$$

Solved using BCTS

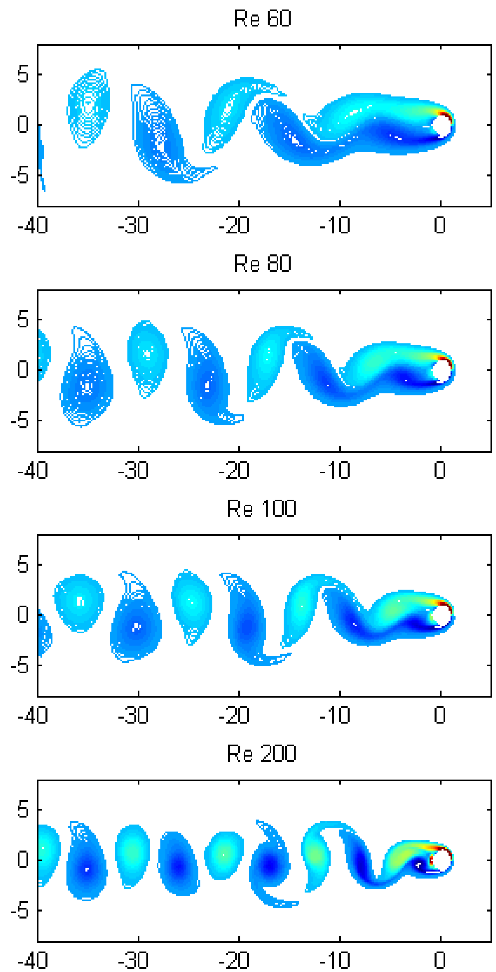
$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial \Psi}{\partial r}\right) + \frac{1}{r^2}\frac{\partial^2 \Psi}{\partial \theta^2} = -\omega$$

Solved using TDMA + FFT

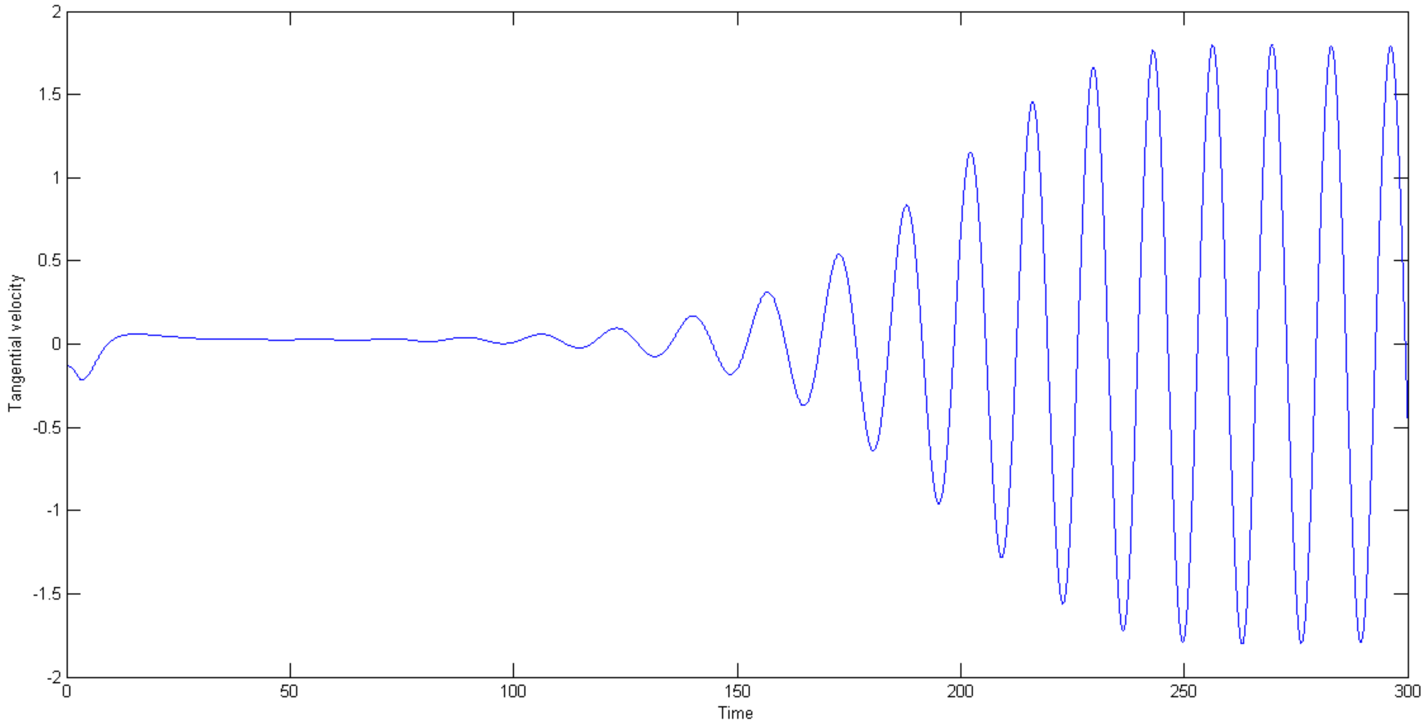
Variation in Strouhal number with Re

Reynolds number	Strouhal number	Strouhal number theory
60	0.133	0.135
80	0.152	0.152
100	0.168	0.166
200	0.196	0.194

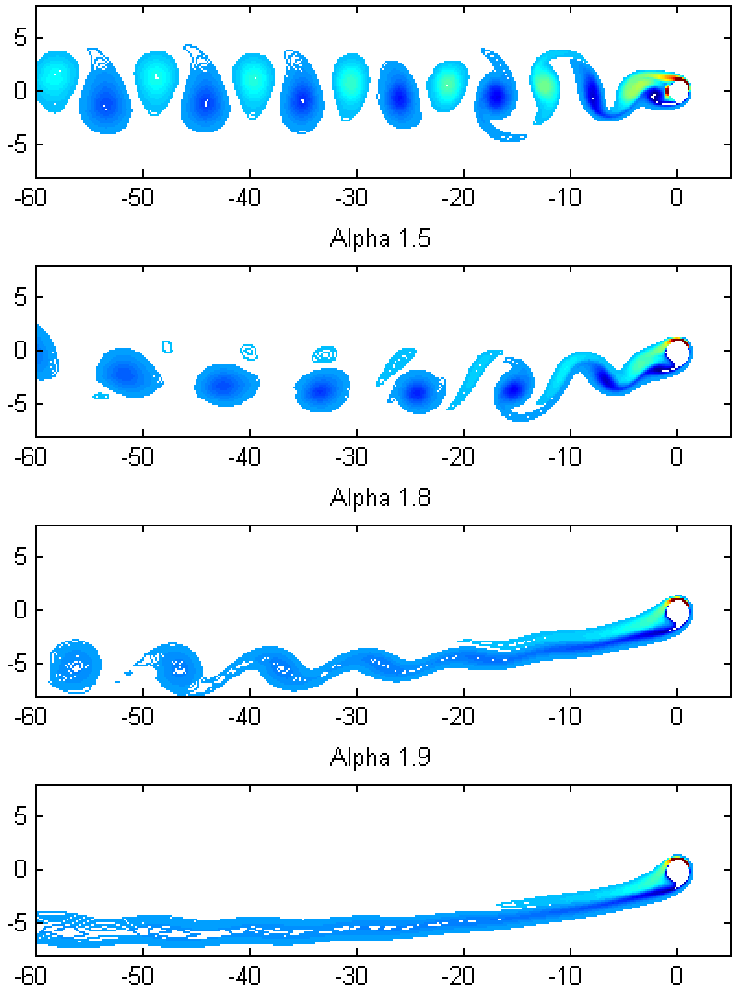
Vortex shedding patterns for various Re



Vortex shedding frequency for Re = 80



Vortex shedding patterns for Re = 200



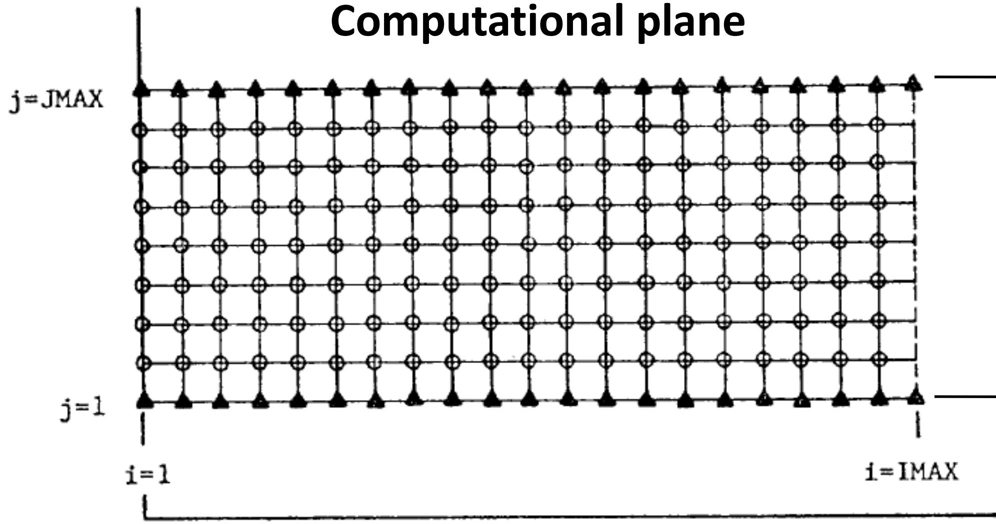
Alpha = V_{θ}/U_{∞}

Adding roughness

- Map a physical domain to Cartesian domain
- Coordinates of boundaries as BCs
- Solve equations using SOR method
- Initial guess must be close to final values
- Solve the pdes in the computational plane

Adding roughness

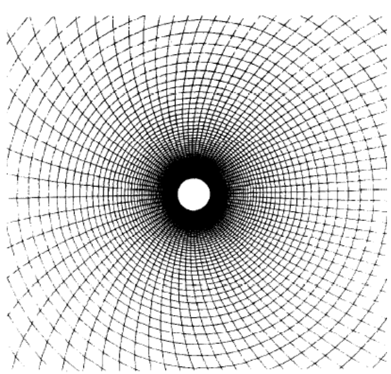
Computational plane

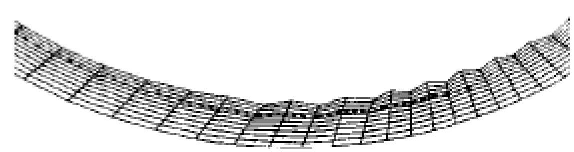


Inner boundary

Outer boundary

Physical plane





Equations for transformation

Generate coordinates

$$\alpha x_{\xi\xi} - 2\beta x_{\xi\eta} + \gamma x_{\eta\eta} = 0$$

$$\alpha y_{\xi\xi} - 2\beta y_{\xi\eta} + \gamma y_{\eta\eta} = 0$$

$$\alpha \equiv x_\eta^{\,2} + y_\eta^{\,2}$$

$$\beta \equiv x_\xi x_\eta + y_\xi y_\eta$$

$$\gamma \equiv x_\xi^{\,2} + y_\xi^{\,2}$$

Solve pde in new domain

$$\omega_t + (\psi_\eta \omega_\xi - \psi_\xi \omega_\eta)/J = (\alpha\omega_{\xi\xi} - 2\beta\omega_{\xi\eta}$$

$$+ \gamma\omega_{\eta\eta} + \sigma\omega_\eta + \tau\omega_\xi)/J^2 R$$

$$\alpha\psi_{\xi\xi} - 2\beta\psi_{\xi\eta} + \gamma\psi_{\eta\eta} + \sigma\psi_\eta + \tau\psi_\xi = -J^2\omega$$

Things to do

- Validate transformation using a perfect cylinder shape
- Run simulations for cylinder with roughness
- Run simulations for rotating cylinder with roughness

Citations

- Defining a Strouhal–Reynolds number ….. circular cylinder
  - *C. H. K. Williamson (1988)*
- Flow past a rotating cylinder
  - *Sanjay Mittal (2003)*
- Computation of high Reynolds number flow around a circular cylinder with surface roughness
  -*T Kawamura (1985)*
- A High Order Explicit Method for the Computation of Flow About a Circular Cylinder
  - *C. R. Anderson (1996)*
- Numerical Solutions of Flow Past a Circular Cylinder at Reynolds Numbers up to 160
  - *J Park (1998)*
- Boundary-fitted curvilinear coordinate systems for solution of PDEs
  – *J. Thomson F. Thames (1977)*