Understanding shock motion through a simple model

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Outline

- Shock Turbulence Interaction (STI)
- Literature
- Motivation
- Theory
- Shock speed formulation
- Geometrical perspective
- Numerical implementation
- Preliminary results
- Advantages and issues
- Summary

Shock wave Turbulence Interaction

Supersonic/Hypersonic flows - High Re

Shock waves

Turbulent flow-field

Interaction

- Shock distortion
- Turbulence amplification

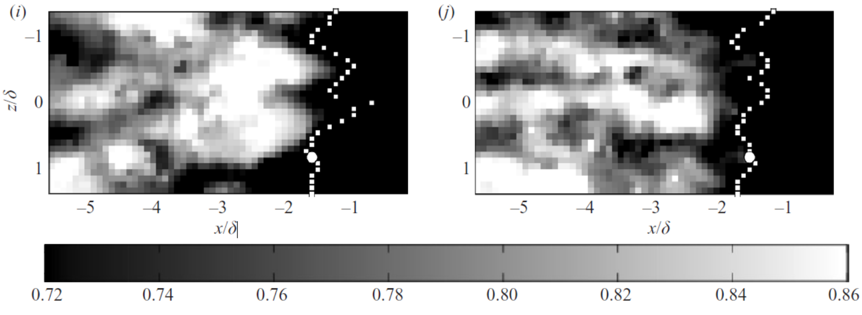
Need for study

SBLI cases

- Engine inlet design
- External aerodynamics
- Design and optimization of control surfaces

Earlier works

Experiments

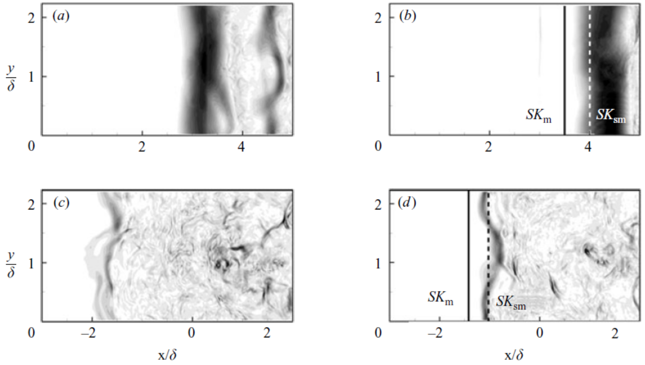


Ganapathisubramani et al., JFM, 2009

Shock motion due to upstream boundary layer

Earlier works - Contd.

Computations

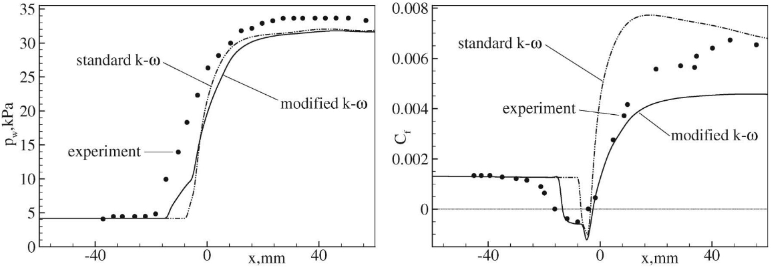


Wu and Martin, JFM, 2008

Shock motion due to downstream separated flow

Motivation

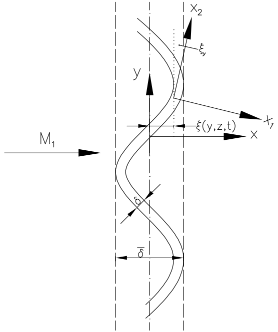
- Unsteady shock motion - size of separation bubble, heat transfer rate, surface pressure distribution.



Pasha and Sinha, IJCFD, 2008

Understand physics of shock unsteadiness via shock speed estimation

Governing equations



Schematic of distorted shock wave

Coordinate transformation

$$x_1 = x - \xi - y\xi_y - z\xi_z$$
$$x_2 = x\xi_y + y$$
$$x_3 = x\xi_z + z$$

Velocities in transformed reference frame

$$u_1 = u - \xi_t - v\xi_y - w\xi_z \simeq \bar{u} + u' - \xi_t$$
$$u_2 = u\xi_y + v \simeq \bar{u}\xi_y + v'$$
$$u_3 = u\xi_z + w \simeq \bar{u}\xi_z + w'$$

Momentum conservation equation

$$\frac{\partial \rho u_1}{\partial t} + \frac{\partial \rho u_1 u_1}{\partial x_1} + \frac{\partial \rho u_1 u_2}{\partial x_2} + \frac{\partial \rho u_1 u_3}{\partial x_3} = -\frac{\partial p}{\partial x_1} + \frac{\partial \sigma_{11}}{\partial x_1} + \frac{\partial \sigma_{12}}{\partial x_2} + \frac{\partial \sigma_{13}}{\partial x_3}$$

Scale analysis

Convective vs. temporal

$$\frac{\partial}{\partial t} \sim \frac{U}{l} \ll U\frac{\partial}{\partial x_1} \sim \frac{U}{\delta}$$

Streamwise vs. transverse

$$\frac{\partial}{\partial x_1} \sim \frac{1}{\delta} \gg \frac{\partial}{\partial x_2} \sim \frac{\partial}{\partial x_3} \sim \frac{1}{l}$$

Viscous vs. inviscid

$$\frac{\partial \rho u_1 u_1}{\partial x_1} \sim \frac{\overline{\rho_u} UU}{\delta} \sim \sigma_{11} \sim \frac{U\overline{\mu_u}}{\delta^2}$$
$$\frac{\overline{\rho_u} U\delta}{\overline{\mu_u}} \sim O\left(1\right)$$

Linearized momentum equation

Reduced momentum equation

$$\frac{\partial \rho u_1 u_1}{\partial x_1} + \frac{\partial p}{\partial x_1} = 0$$

Inertial frame of reference

$$\frac{\partial \rho (u - \xi_t)(u - \xi_t)}{\partial x} + \frac{\partial p}{\partial x} = 0$$

Mean equation

$$\frac{\partial \overline{\rho u u}}{\partial x} + \frac{\partial \overline{p}}{\partial x} = 0$$

Linearized equation

$$\bar{u}\frac{\partial u'}{\partial x} + (u' - \xi_t)\frac{\partial \bar{u}}{\partial x} + \frac{1}{\bar{\rho}}\frac{\partial p'}{\partial x} - \frac{\rho'}{\bar{\rho}^2}\frac{\partial \bar{p}}{\partial x} = 0$$

Euler momentum equation

1D inviscid Euler momentum equation

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + \frac{1}{\rho}\frac{\partial p}{\partial x} = 0$$

Mean equation

$$\frac{\partial \bar{u}}{\partial t} + \bar{u}\frac{\partial \bar{u}}{\partial x} + \frac{1}{\bar{\rho}}\frac{\partial \bar{p}}{\partial x} = 0$$

Linearized Euler equation

$$\frac{\partial u'}{\partial t} + \bar{u}\frac{\partial u'}{\partial x} + u'\frac{\partial \bar{u}}{\partial x} + \frac{1}{\bar{\rho}}\frac{\partial p'}{\partial x} - \frac{\rho'}{\bar{\rho}^2}\frac{\partial \bar{p}}{\partial x} = 0$$

Shock speed

Linearized governing equation for shock

$$\bar{u}\frac{\partial u'}{\partial x} + (u' - \xi_t)\frac{\partial \bar{u}}{\partial x} + \frac{1}{\bar{\rho}}\frac{\partial p'}{\partial x} - \frac{\rho'}{\bar{\rho}^2}\frac{\partial \bar{p}}{\partial x} = 0$$

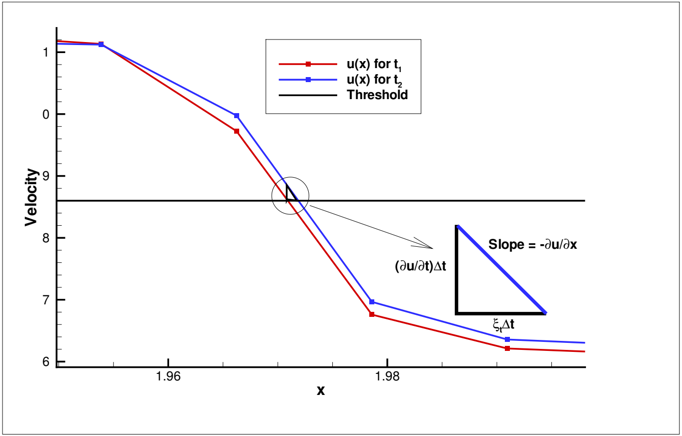
Linearized Euler equation

$$\frac{\partial u'}{\partial t} + \bar{u}\frac{\partial u'}{\partial x} + u'\frac{\partial \bar{u}}{\partial x} + \frac{1}{\bar{\rho}}\frac{\partial p'}{\partial x} - \frac{\rho'}{\bar{\rho}^2}\frac{\partial \bar{p}}{\partial x} = 0$$

Comparison

$$\frac{\partial u'}{\partial t} \approx -\xi_t\frac{\partial \bar{u}}{\partial x}$$

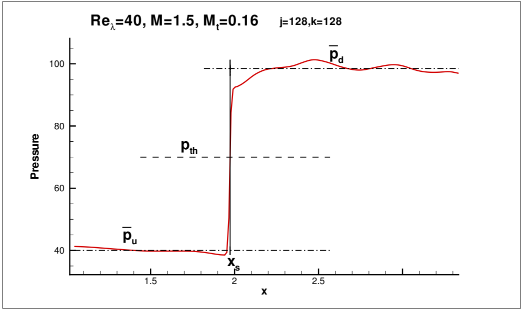
Geometric perspective



Velocity profile at time instances t_1 and t_2

$$-\frac{\partial u}{\partial x} = \frac{\frac{\partial u}{\partial t}.\Delta t}{\xi_t.\Delta t}$$

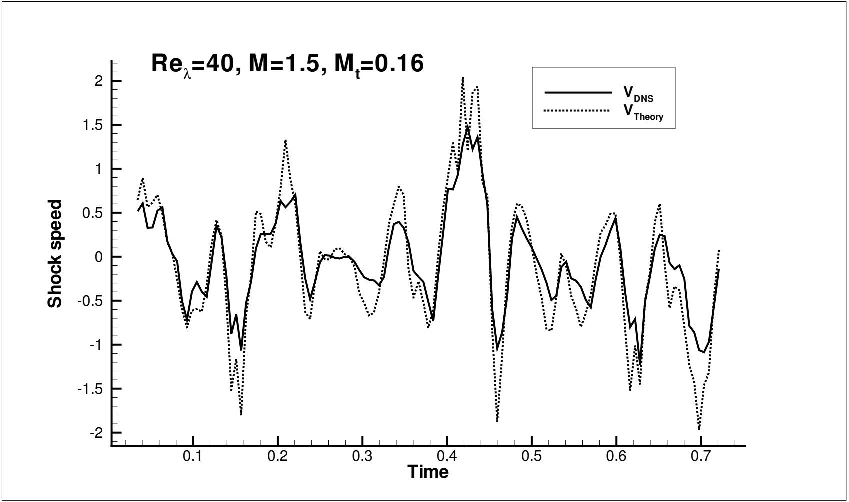
Shock location



Instantaneous pressure profile from DNS data

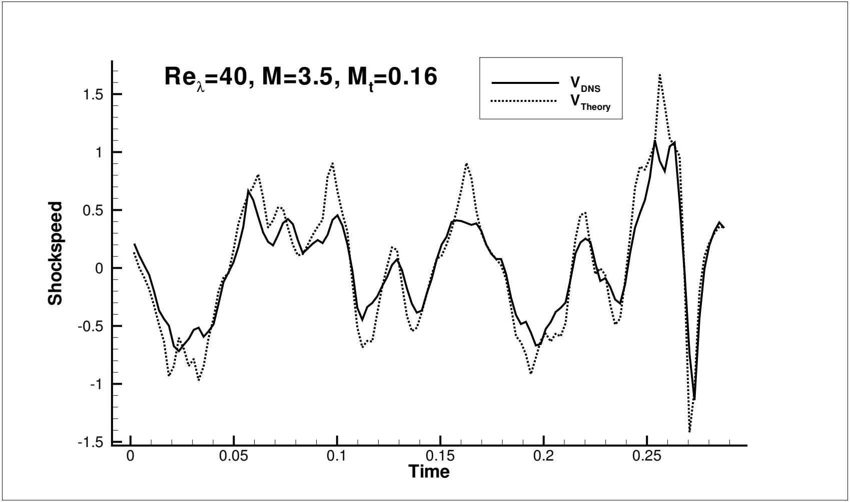
- Time averaged pressure profile - pressure threshold
- Linear interpolation - shock location for DNS shock speed
- Most negative dilatation point $\frac{\partial u}{\partial x}$ for theoretical shock speed

Preliminary results



Time varying shock speed plot for $Re_\lambda = 40,\ M = 1.5,\ M_t = 0.16$ DNS data

Preliminary results - Contd.



Time varying shock speed plot for $Re_\lambda = 40,\ M = 3.5,\ M_t = 0.16$ DNS data

Advantages and Issues

Numerical issues

Identification of exact grid points

Advantages

Simple formulation - physical

Physical issues

Non-linear and viscous terms

Uses

Quantify shock unsteadiness

Current & Future work

- Use mass equation to derive shock speed
- Modifying expression to provide actual speed
- Identify the reasons for discrepancies
- Identify whether the same can be represented by 1D simulation
- Analysis and comparison of 1D case with full DNS - forced broadband fluctuations
- Analysis of time scales of shock motion
- Cause of shock unsteadiness
- Consistent and efficient way to represent shock unsteadiness

Summary

- Previous works on STI and motivation
- Governing equations in shock frame of reference
- Linearized Euler equations in inertial frame of reference
- Comparison of Euler and shock governing equations — Shock speed
- Geometrical analogy of formulation
- Numerical implementation of formulation
- Shock location and comparison of two shock speeds — DNS and theory
- Advantages and issues in formulation

Thank You!