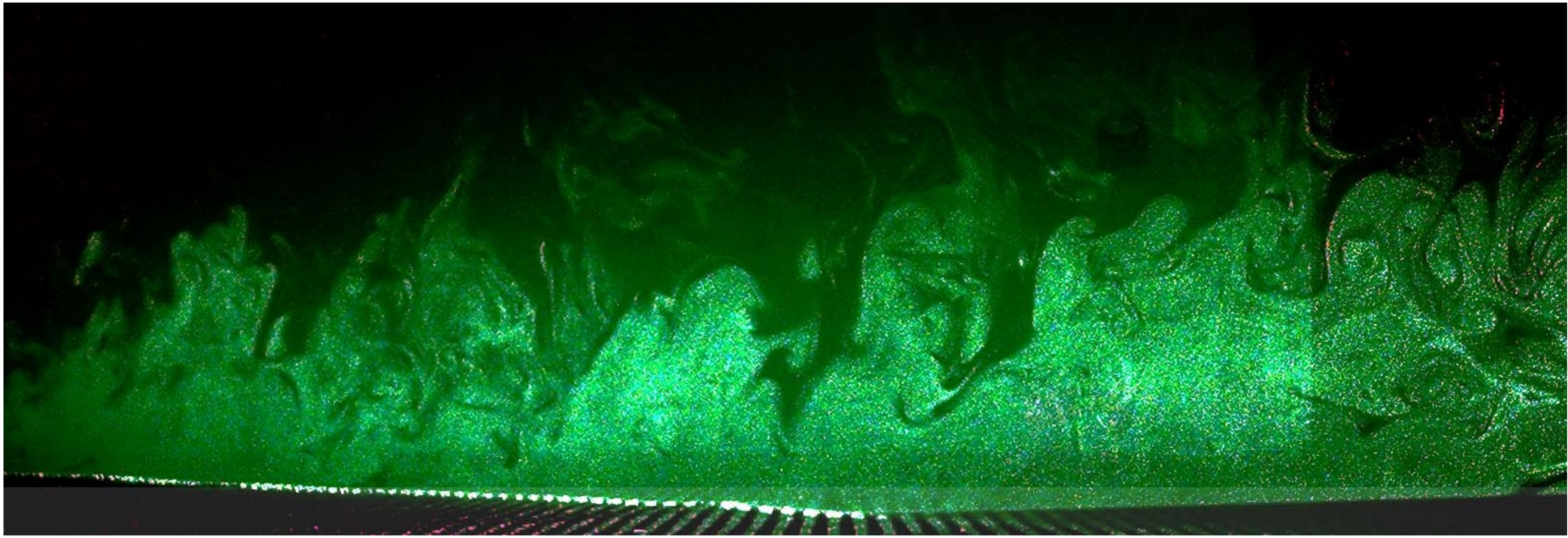
Morkovin Hypothesis and Modeling of Compressible Flows

2nd TIFR – IITB meet



PIV, Olive oil seed particles.

Fernando Porté-Agel, U. Minn.

Theme

- Morkovin Hypothesis
- Shock and the hypothesis validity
- Implications on turbulence modeling

Morkovin's hypothesis

Introduction

- Mark Vladimir Morkovin – 1962 : Effects of Compressibility on Turbulent Flows," Mecanique de la Turbulence

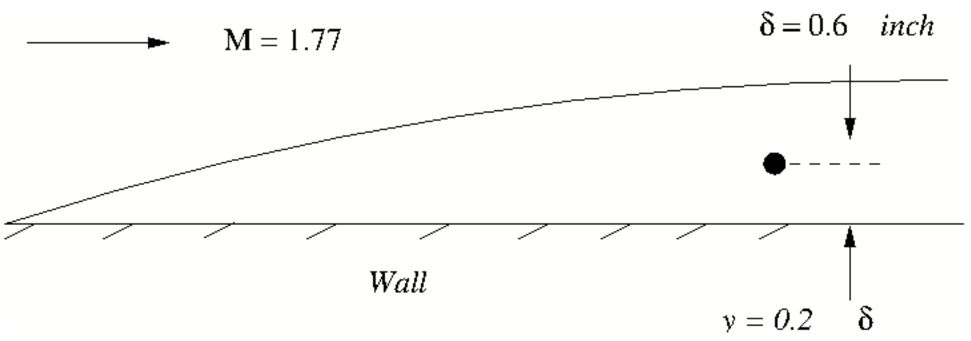
The essential dynamics (Eddy pattern) of wall bounded compressible flow will follow the incompressible pattern.

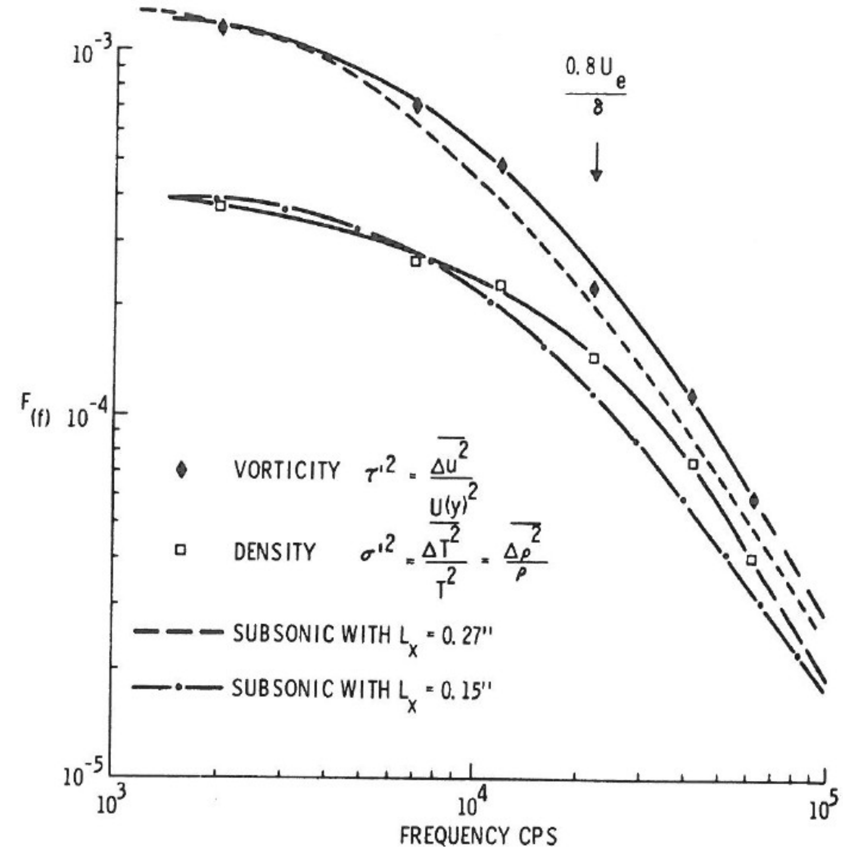
- The eddy pattern will be altered as per mean density change

- Density fluctuations has very less effect on turbulence provided $\rho' / \bar{\rho} << 1$ (generally the case)

Firm Base for similarity

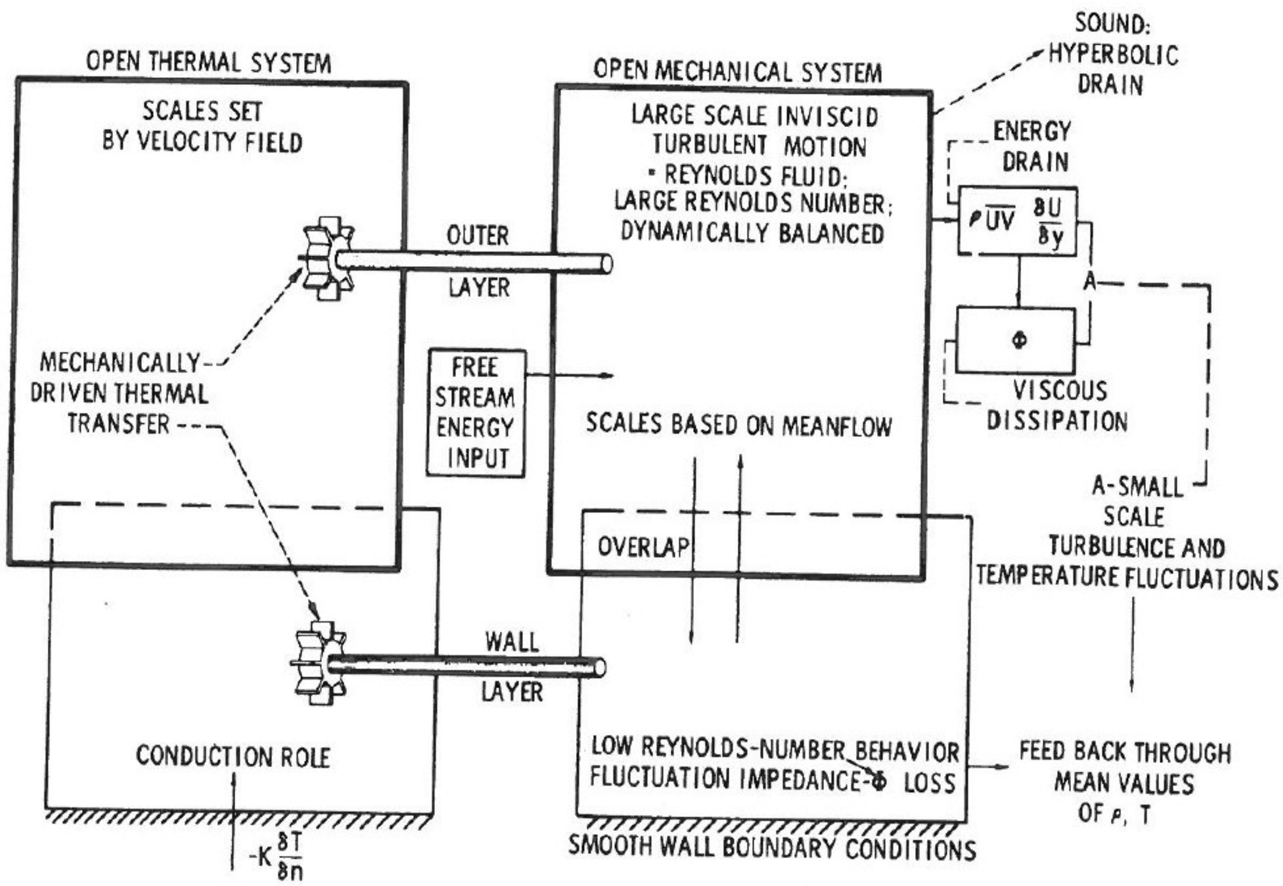
- Large scale eddies govern turbulence dynamics – Low speed and high speed
- Coupling at small scales





schematic

Schematic of coupling



Governing Equations

- For compressible B.L. Flow stationary in the mean, equations reduce to

$$\frac{\partial}{\partial x}(\overline{\rho}\widetilde{u}) + \frac{\partial}{\partial y}(\overline{\rho}\widetilde{v}) = 0$$

$$\overline{\rho}\widetilde{u}\frac{\partial \widetilde{u}}{\partial x} + \overline{\rho}\widetilde{v}\frac{\partial \widetilde{u}}{\partial y} = -\frac{\partial \overline{p}}{\partial x} + \frac{\partial}{\partial y}\left(\widetilde{\sigma}_{xy} - \overline{\rho}\widetilde{u''v''}\right)$$

$$\overline{\rho}\widetilde{u}\frac{\partial \widetilde{H}}{\partial x} + \overline{\rho}\widetilde{v}\frac{\partial \widetilde{H}}{\partial y} = \frac{\partial}{\partial y}\left[\frac{\overline{k}_T}{c_p}\frac{\partial \widetilde{H}}{\partial y} - \overline{\rho}\widetilde{v''H''} + \mu\left(1 - \frac{1}{Pr}\right)\frac{\partial}{\partial y}\left(\frac{\widetilde{u}^2}{2} + \frac{\widetilde{u''^2}}{2}\right)\right]$$

- Zero mean pressure gradient ; Domination of turbulent processes

$$\widetilde{H} = b_0\widetilde{u} + b_1 \qquad \widetilde{H} - \widetilde{H}_w = c_p\widetilde{T}_t - c_p\widetilde{T}_{tw} = Pr_w\left(\frac{\widetilde{q}_y}{\widetilde{\sigma}_{xy}}\right)\Bigg|_w \widetilde{u}.$$

Strong Reynolds Analogy

$$\widetilde{H} - \widetilde{H}_w = Pr_w \left(\frac{\widetilde{q}_y}{\widetilde{\sigma}_{xy}} \right) \bigg|_w \widetilde{u}.$$

$$H'' = Pr_w \left(\frac{\widetilde{q}_y}{\widetilde{\sigma}_{xy}} \right) \bigg|_w u''.$$

Adiabatic Flow

$$\boxed{T'_t = 0}$$

$$T' + \left(\frac{\overline{u}}{c_p} \right) u' = 0$$

$$\frac{T'}{\overline{T}} = -(\gamma - 1)\overline{M}^2 \left(\frac{u'}{\overline{u}} \right)$$

Strong form

$$\frac{\sqrt{\overline{T'^2}}}{\overline{T}} \approx (\gamma - 1)\,\overline{M}^2 \frac{\sqrt{\overline{u'^2}}}{\overline{u}}$$

Temp. Vel. Correlation

$$R_{u'T'} \equiv \frac{\overline{u'T'}}{\sqrt{\overline{u'^2}}\sqrt{\overline{T'^2}}}$$

$$T' + \frac{\overline{u}}{C_p}u' = 0$$

$$\sqrt{\overline{T'^2}} = \frac{\overline{u}}{C_p}\sqrt{\overline{u'^2}}$$

$$\boxed{R_{u'T'} = -1}$$

Turbulent Prandtl number

- Represents the ratio of turbulent momentum transfer to turbulent heat transfer

$$\overline{T}_t \approx \overline{T} + \frac{\overline{u}^2}{2c_p}$$

$$Pr_t = \frac{-\overline{\rho}\overline{u'v'}(\partial\overline{T}/\partial y)}{-\overline{\rho}\overline{v'T'}(\partial\overline{u}/\partial y)}$$

$$Pr_t = \left[1 - \frac{\overline{v'T'_t}}{\overline{v'T'}}\right]\left[1 - \frac{\partial\overline{T}_t}{\partial\overline{T}}\right]^{-1}$$

$$\overline{v'T'} = -\left(\frac{\overline{u}}{c_p}\right)\overline{u'v'} + \overline{v'T'_t}$$

$$\boxed{Pr_t = 1}$$

Reynolds Averaged Equation

- The governing equations

$$\frac{\partial \bar{\rho}}{\partial t} + \frac{\partial(\bar{\rho}\tilde{u}_i)}{\partial x_i} = 0$$

$$\frac{\partial}{\partial t}(\bar{\rho}\tilde{u}_i) + \frac{\partial}{\partial x_j}(\bar{\rho}\tilde{u}_i\tilde{u}_j) = -\frac{\partial \bar{p}}{\partial x_i} + \frac{\partial}{\partial x_j}\left[\bar{t}_{ij} - \overline{\rho u_j'' u_i''}\right]$$

$$\frac{\partial \bar{E}}{\partial t} + \frac{\partial}{\partial x_j}\left[\tilde{u}_j \bar{E}\right] = -\frac{\partial}{\partial x_j}(\bar{p}\tilde{u}_j) + \frac{\partial}{\partial x_j}\left[-\bar{q}_j - \overline{\rho u_j'' h''}\right] + \text{Other terms}$$

Unclosed terms

$\overline{\rho u'' v''}$ $\overline{\rho v'' v''}$ $\overline{\rho u'' h''}$ $\overline{\rho v'' h''}$

Reynolds stress tensor

Turbulent energy flux

Modeling

Turbulent energy flux

$$\overline{\rho u_i'' h''} = -\frac{\mu_T C_p}{Pr_T}\frac{\partial \tilde{T}}{\partial x_i}$$

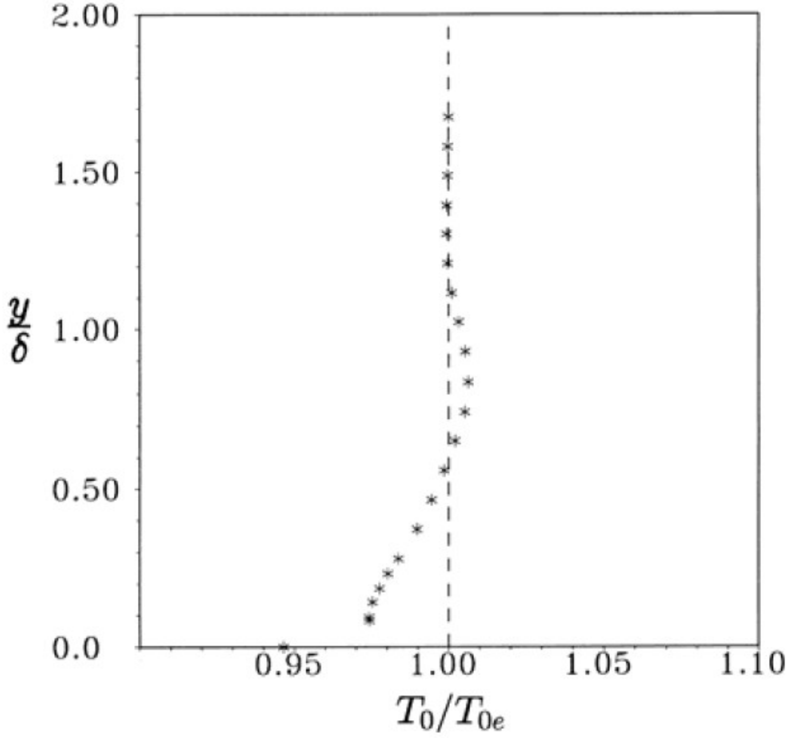
For boundary layer flows, constant $Pr_T \sim 0.89$ is used for modeling

Trends

$$\boxed{T_t' = 0}$$

Adiabatic boundary layer flat plate

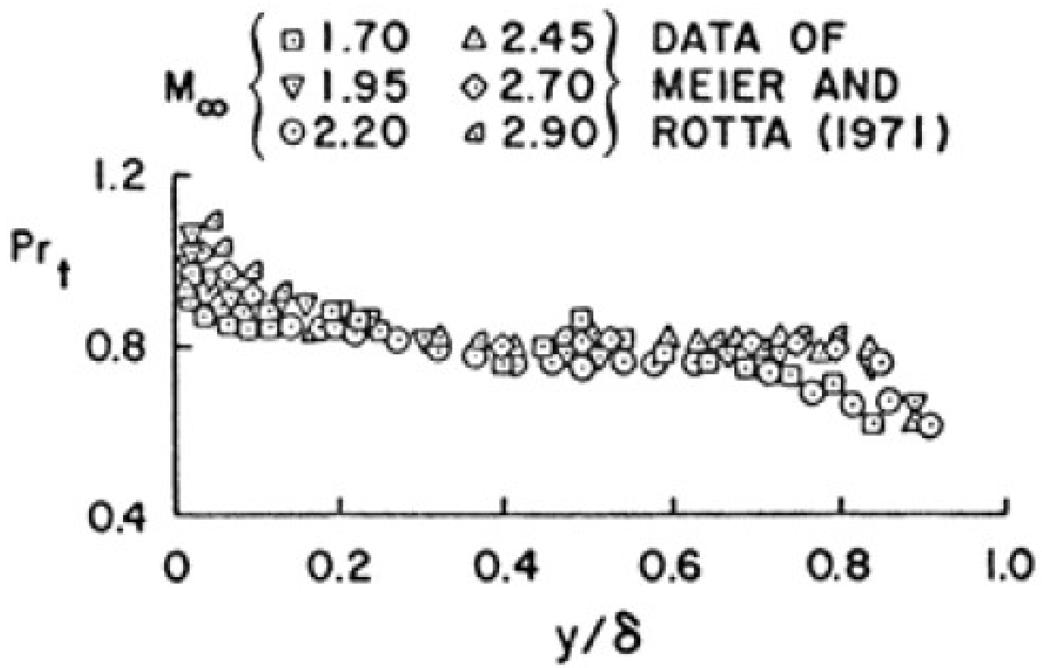
M = 2.3; Re_θ = 5500



Laurent (1996)

Trends

$$\boxed{Pr_t = 1}$$



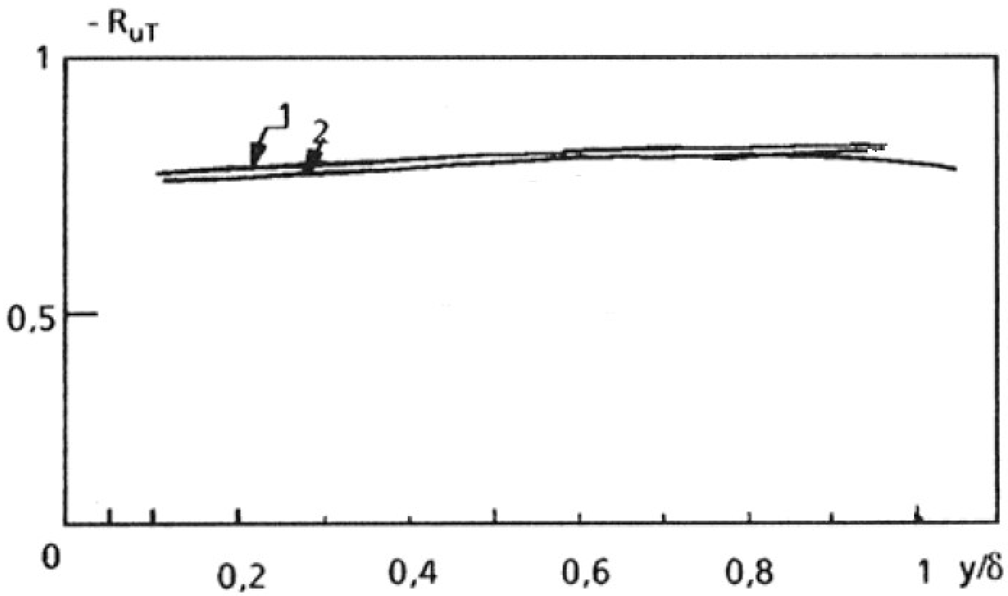
- Turbulent transport of heat as compared to momentum, decreases rapidly at the wall

Trends

$$\boxed{R_{u'T'} = -1}$$

M = 2.32; Re_θ = 5650;

M = 1.73; Re_θ = 5700

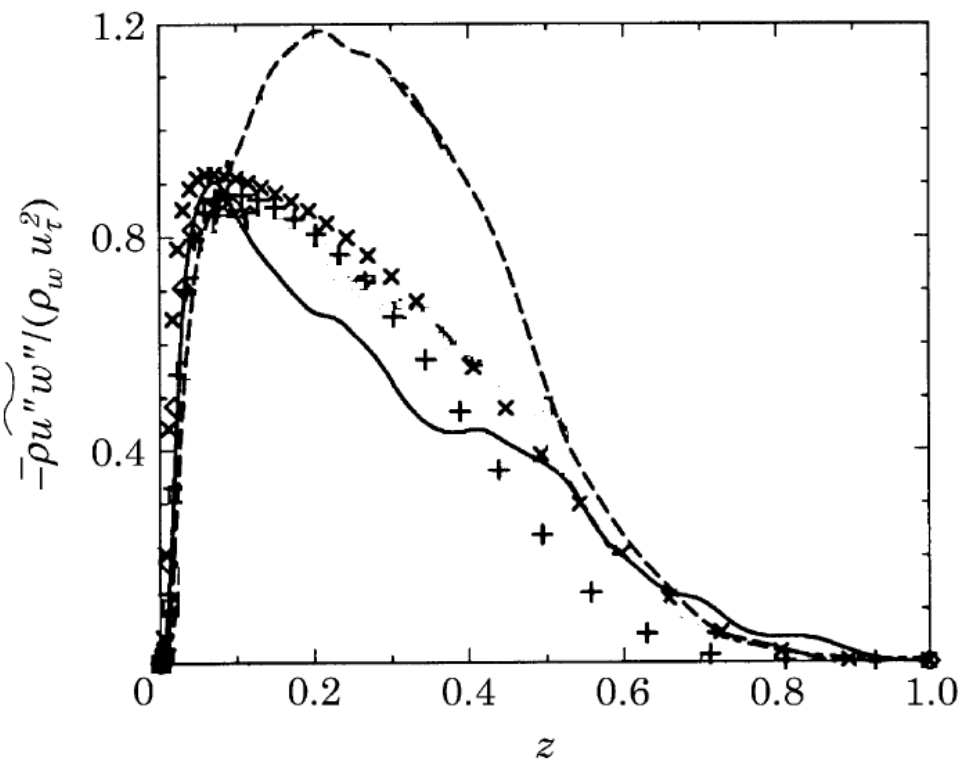


Elena and Gaviglio (1993)

Trends

+ ,-: M = 3; Re_θ = 3028; Re_θ (Incomp)= 1456

X , - -: M = 4.5; Re_θ = 3305; Re_θ (Incomp)= 1032



Interim Summary:

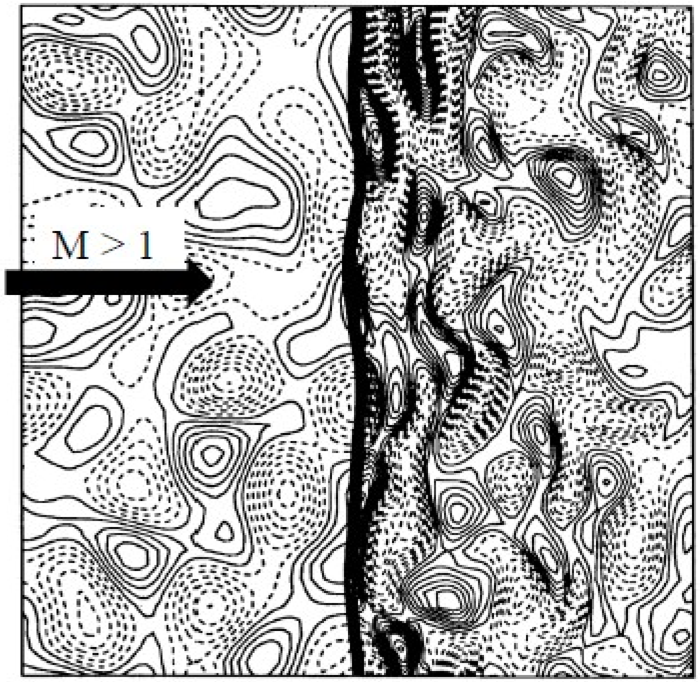
Wall-bounded flows with adiabatic boundary: $R_{u'T'} = -1$, $Pr_t = 1$

Same turbulence models apply to compressible wall bounded flows with mean density correction

What happens when shocks exist ?

Canonical case

- Incoming turbulence (Vorticity and entropy)
- 1D mean uniform flow
- Normal shock
- Shock distorts, turbulence amplifies



DNS of Lee et al. (1993)

DNS of shock-turbulence

- Cases considered by Mahesh et al.

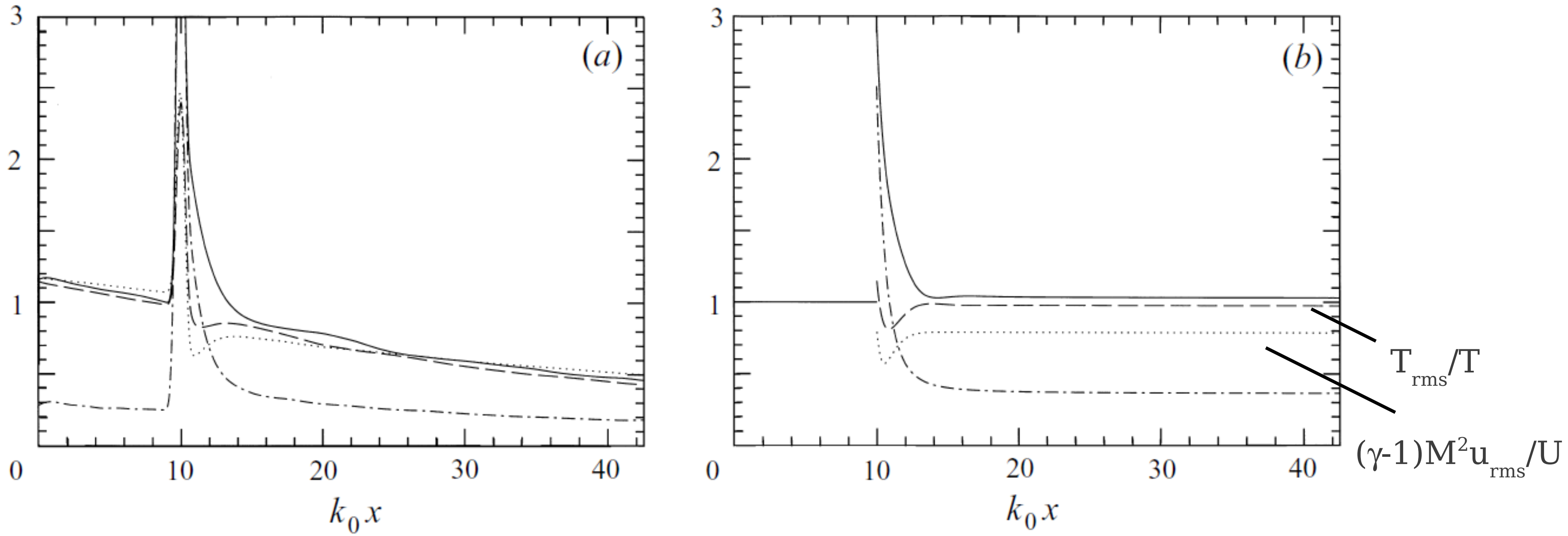
	Case 1.29	Case 1.8
M_1	1.29	1.8
R_λ	19.1	19.5
M_t	0.14	0.18

- Zero upstream pressure fluctuations

- Upstream Morkovin hypothesis valid

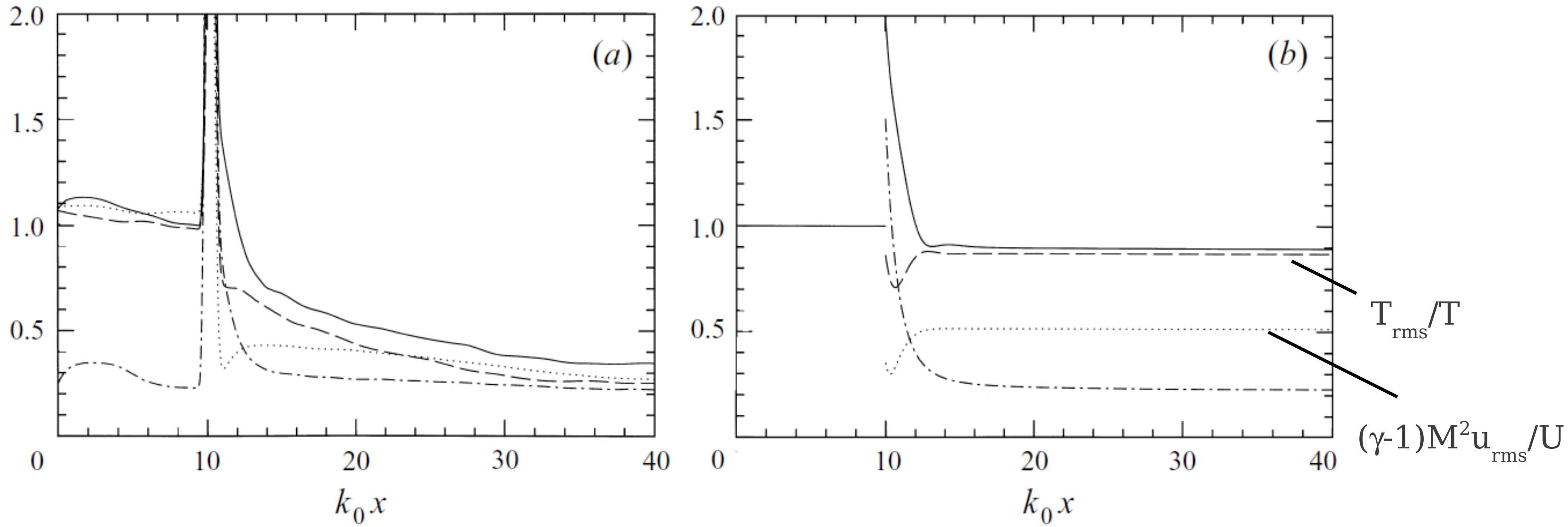
$$\frac{\rho'}{\overline{\rho}} = -\frac{T'}{\overline{T}} = (\gamma - 1)M^2\frac{u'}{U}$$

DNS : M - 1.29



$R_{u'T'} = -1 \longrightarrow R_{u'T'} = -0.54$

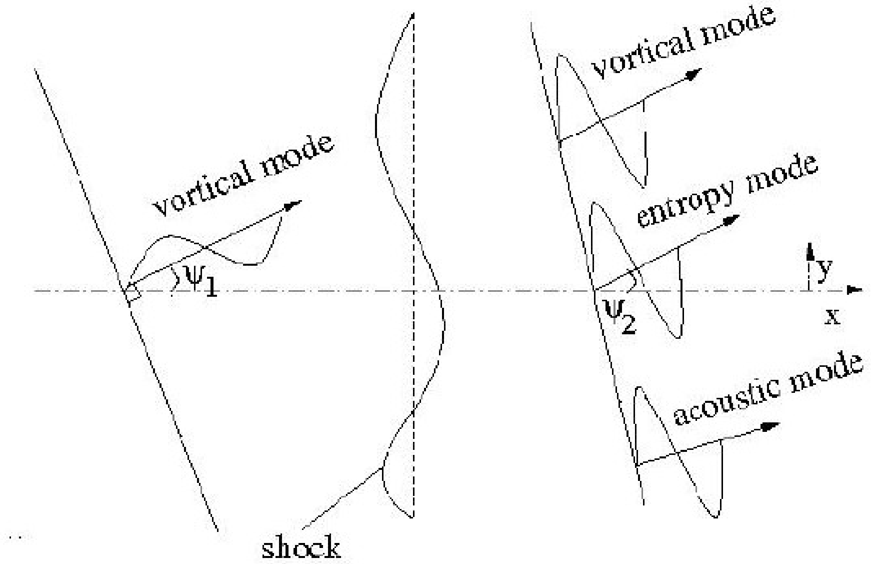
DNS : M - 1.80



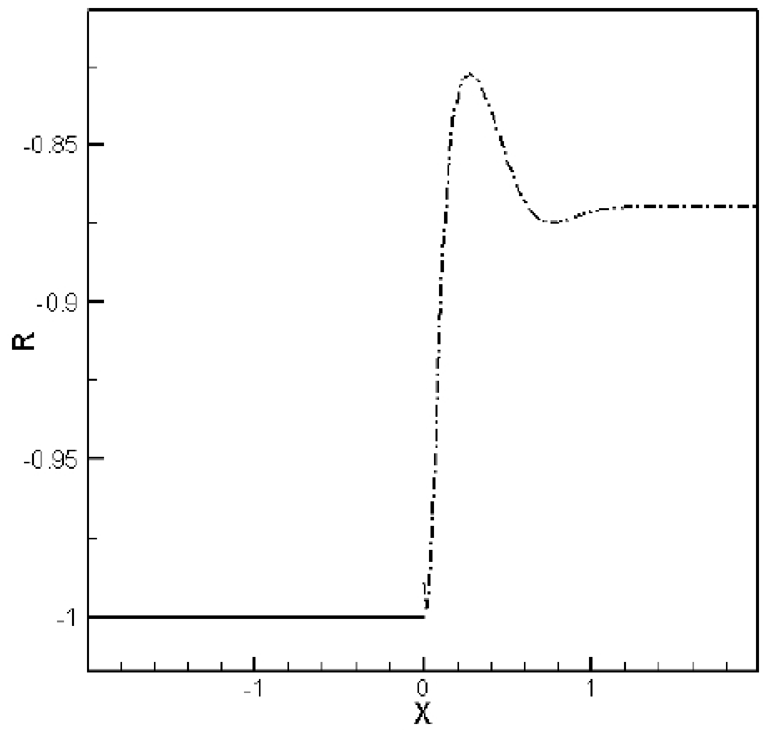
$R_{u'T'} = -1 \longrightarrow R_{u'T'} = -0.22$

Linear Interaction Analysis

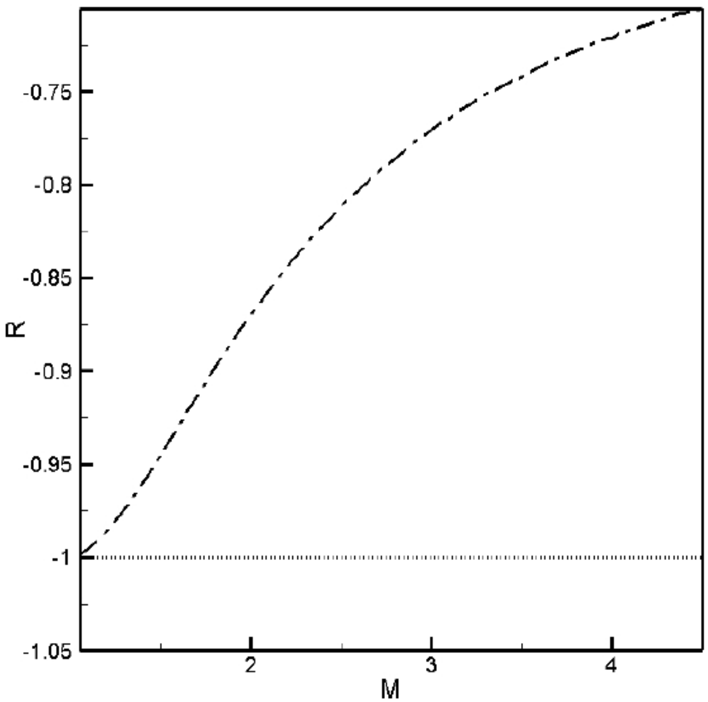
- Theory : Ribner, 1953
- Each independent interaction
- Downstream turbulence = Superposition of several linear interactions
- Linear interaction analysis forms the limiting case of low Mt and high Re turbulence



LIA – $R_{u'T'}$



M=2; $R_{u'T'}$ variation along x



$R_{u'T'}$ variation far-field for varying upstream Mach

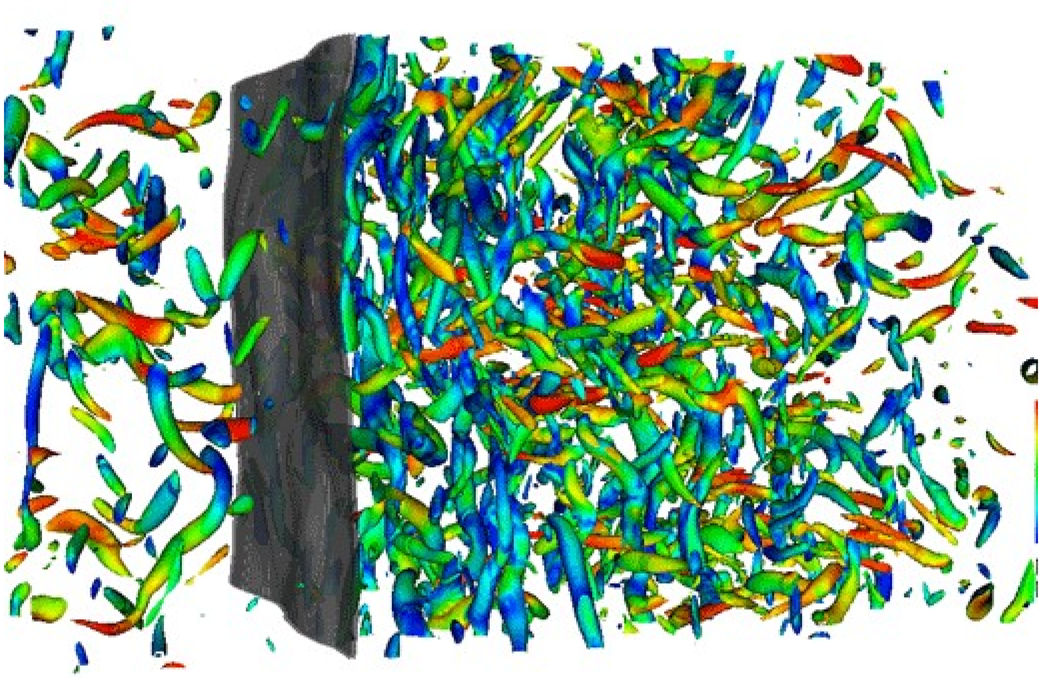
Interim Summary:

Shock alters the correlation coefficient, $R_{u'T'}$

A simpler case of shock-turbulence : $R_{u'T'}$?

Shock-turbulence – Canonical case

- Upstream turbulence – Purely vortical
- Theoretical tool – Linear Invscid Analysis
- Extensive DNS data available



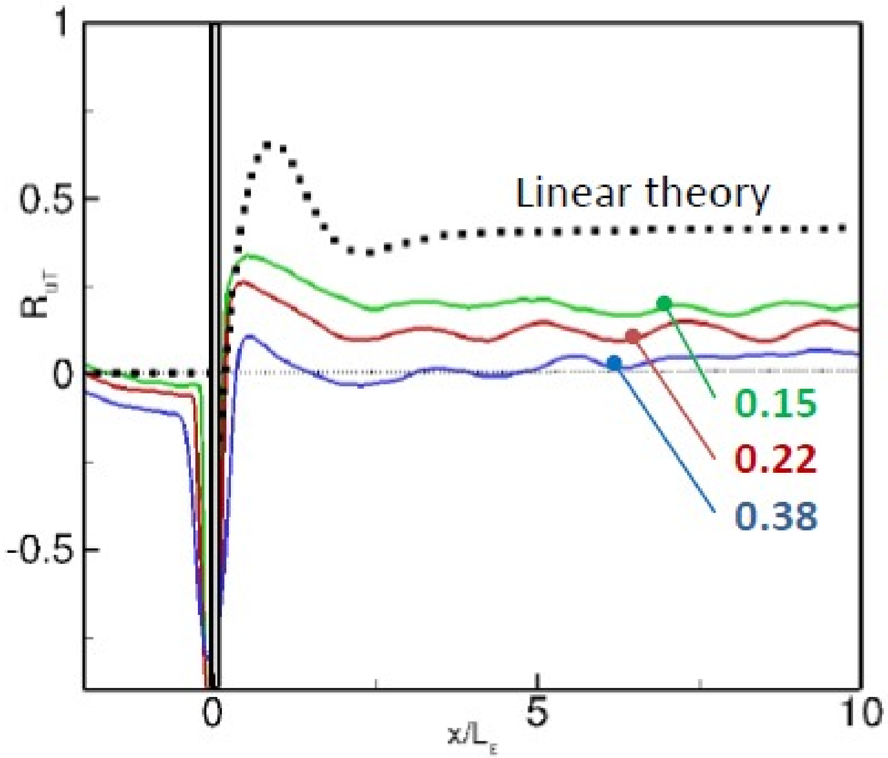
Johan Larsson

DNS data

Re_λ	M	M_t
39	1.05	0.05
38	1.28	0.15
39	1.28	0.22
38	1.28	0.26
38	1.28	0.31
38	1.50	0.15
39	1.50	0.22
39	1.51	0.31
39	1.51	0.37
39	1.87	0.22
39	1.87	0.31
40	2.50	0.22
40	3.50	0.16
41	3.50	0.23
42	4.70	0.23
42	6.00	0.23
73	1.50	0.14
73	1.50	0.22
72	1.52	0.38
74	3.50	0.15

- Larsson et al. JFM, 2013

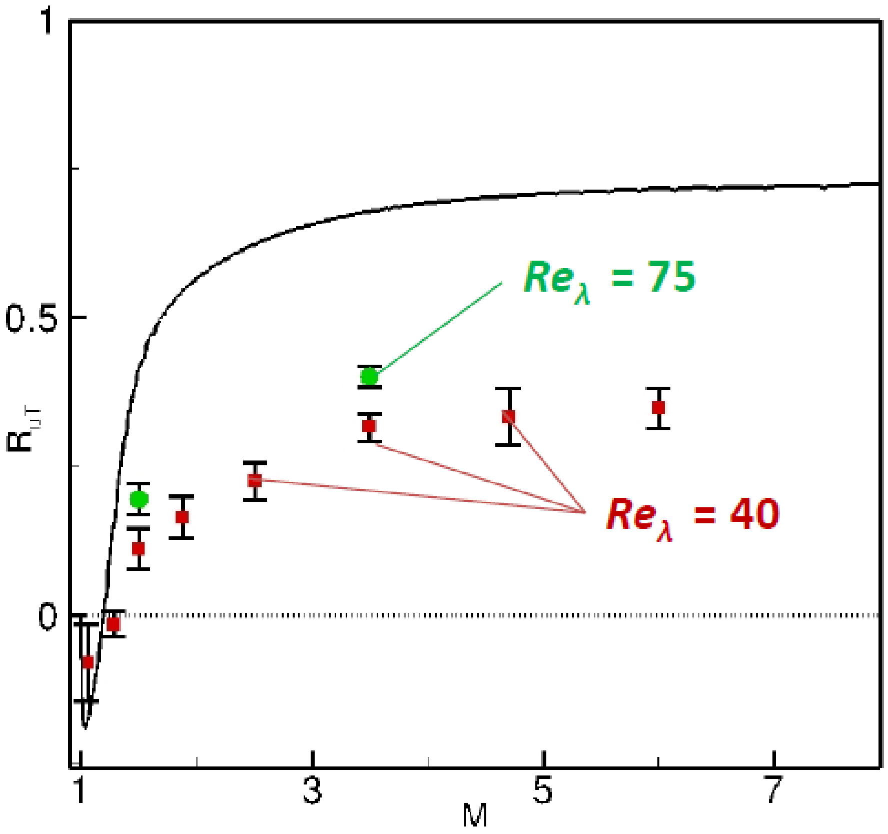
Effect of M_t



M = 1.5 , Re_λ = 75

- RuT plotted for varying x
- Qualitative trend - Transient peak and steady value observed in theory
- Lower Mt case closer to LIA

Effect of Re$_\lambda$



- RuT Vs. M
- Negative RuT for lower Mach numbers
- RuT increases for higher Re – Closer to linear limit

Interim Summary:

When upstream turbulence is purely vortical, a positive $R_{u'T'}$ is obtained behind the shock

Implication on turbulence modeling ?

Modeling

For boundary layers $R_{u'T}=-1$ ➡ $Pr_T \sim 0.9 - 1$

Entropy and vorticity

$\boxed{0>R_{u'T}>-1}$

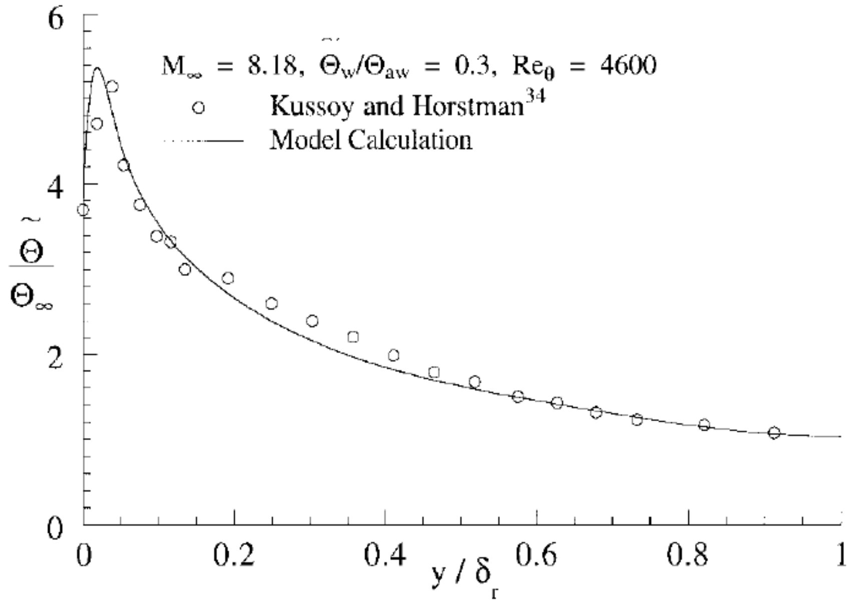
For Shock dominated flow

Pr_T - ? ? ?

$\boxed{1>R_{u'T}>0}$

Vorticity

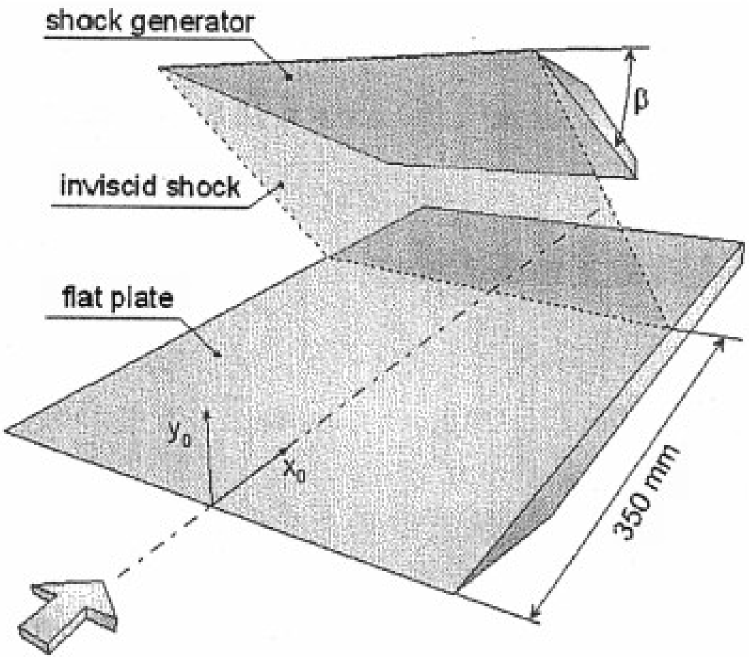
Modeling – Boundary layer

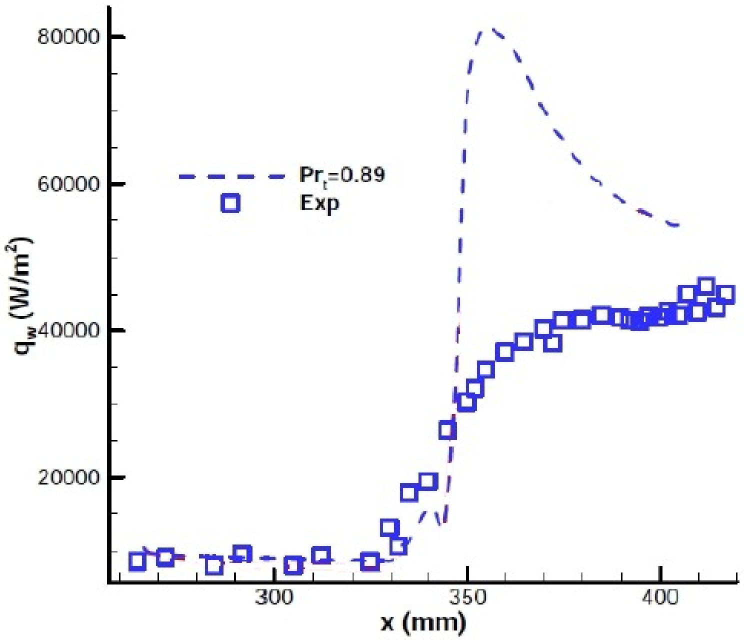


So et al.

Modeling - Shock

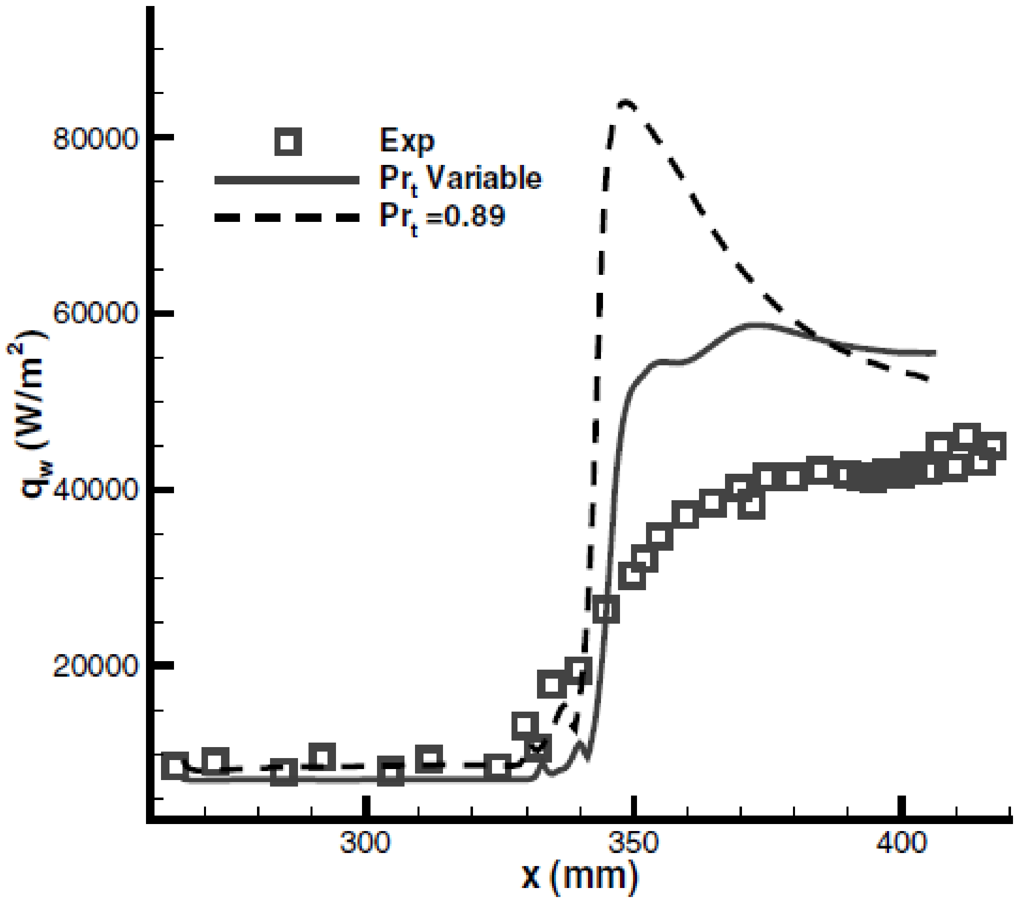
M = 5





Prt?

Variable Pr_T modeling



Interim Summary:

A constant Pr_T model fails in prediction of SBLI flows

Pr_T has to be varied at the shock in order to account effect of discontinuity on turbulent heat flux

Summary :

- Wall-bounded flows with adiabatic boundary: $R_{u'T'} = -1$, $Pr_t = 1$
- Same turbulence models apply to compressible wall bounded flows with mean density correction
- Shock alters the correlation coefficient, Pr_t ,$R_{u'T'}$
- When upstream turbulence is purely vortical, a positive $R_{u'T'}$ is obtained behind the shock
- A constant Pr_T model fails in prediction of SBLI flows
- Variable Pr_T improves heat flux prediction

Can a Physics Based model be developed to account for this phenomenon?

Thank you for listening !