

Modeling shock unsteadiness in shock/turbulence interaction

V. Jagadish Babu

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IIT Bombay

Supervisor
Prof. Krishnendu Sinha

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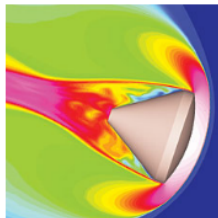


Outline

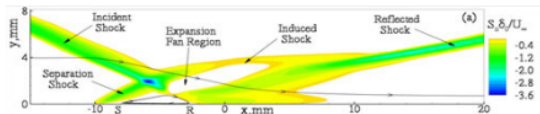
- 1 Shock wave boundary layer interactions
- 2 Shock-turbulence interaction
- 3 Modeling improvement
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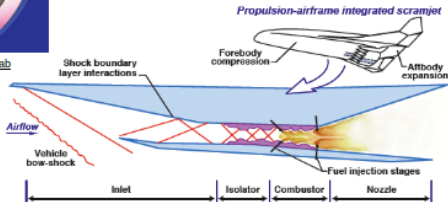
Shock wave boundary layer interactions(SBLI)



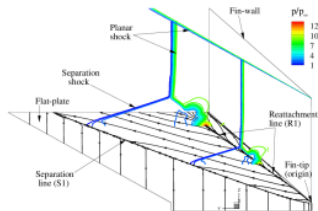
Hypersonic-cfd lab



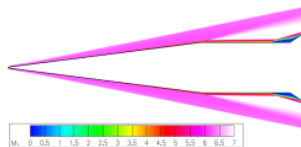
Hypersonic-cfd lab



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Hypersonic-cfd lab



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Turbulent transport equations ($k - \epsilon$ model)

$$\bar{\rho} \tilde{u}_j \frac{\partial k}{\partial x_j} = -\widetilde{\bar{\rho} u_i'' u_j''} \frac{\partial \tilde{u}_i}{\partial x_j} - \bar{\rho} \epsilon_s + \frac{\partial}{\partial x_j} \left[\left(\mu + \frac{\mu_T}{\sigma_k} \right) \frac{\partial k}{\partial x_j} \right]$$

$$\bar{\rho} \tilde{u}_j \frac{\partial \epsilon_s}{\partial x_j} = -C_{\epsilon 1} \widetilde{\bar{\rho} u'' u''} \frac{\partial \tilde{u}}{\partial x} \frac{\epsilon_s}{k} - C_{\epsilon 2} \bar{\rho} \frac{\epsilon_s^2}{k} + \frac{\partial}{\partial x_j} \left[\left(\mu + \frac{\mu_T}{\sigma_\epsilon} \right) \frac{\partial \epsilon_s}{\partial x_j} \right]$$

$k = \frac{1}{2} \widetilde{u_i'' u_i''}$ is turbulent kinetic energy,

$\widetilde{\bar{\rho} u_i'' u_j''}$ is Reynolds stress tensor,

ϵ_s is solenoidal dissipation rate,

μ is viscosity and μ_T is eddy viscosity,

$C_{\epsilon 1} = 1.35$ and $C_{\epsilon 2} = 1.8$.



Eddy viscosity concept

- Reynolds stress is proportional to strain rate, with turbulent eddy viscosity being a proportionality constant

$$-\overline{\rho u'_i u'_j} = \nu_t \left(\frac{\partial \bar{u}_i}{\partial x_j} + \frac{\partial \bar{u}_j}{\partial x_i} \right)$$

- Model is applicable only for flows where the timescale of turbulence is comparable with that of timescale of mean strain.
- Model fails for non-equilibrium flows (e.g. flows with shocks or strong separation or swirl)
- One and Two - equation models which make use of this hypothesis do not predict correctly.



Some attempts to improve predictions

- Realizability constraint (Liou *et al.*, 2000)
- Compressibility correction (Forsythe *et al.*, 1998)
- Length-scale modification (Coakley *et al.*, 1992)
- Rapid compression correction (Coakley *et al.*, 1992)

Outcome

- These modifications vary from model to model.
- Key physical processes are modeled incorrectly or not included



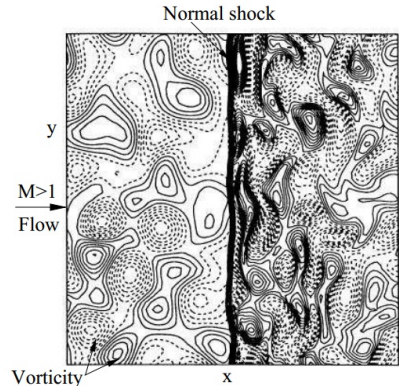
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Shock-turbulence interaction(STI)

- Homogeneous isotropic turbulence passing through a normal shock.
- No separation, streamline curvature, and boundary-layer velocity gradients. Effect of shock on turbulence.
- The mean flow is steady one-dimensional and uniform upstream and downstream of the shock wave.
- The jump across the shock is governed by the Rankine-Hugoniot relations.



Lee *et al.*



Boussinesq hypothesis

$$-\overline{\rho u_i'' u_j''} = 2\mu_T \left(S_{ij} - \frac{1}{3} \frac{\partial \tilde{u}_k}{\partial x_k} \delta_{ij} \right) - \frac{2}{3} \bar{\rho} k \delta_{ij}$$

- Standard $k - \epsilon$ model

$$\mu_T = c_\mu \frac{\bar{\rho} k^2}{\epsilon_s} \quad \Rightarrow \quad \overline{\rho u'' u''} = -\frac{4}{3} \mu_T \frac{\partial \tilde{u}}{\partial x} + \frac{2}{3} \bar{\rho} k$$

- Realizable $k - \epsilon$ model

$$\mu_T = \frac{\sqrt{3c_\mu^0}}{2} \frac{\bar{\rho} k}{\partial \tilde{u} / \partial x} \quad \Rightarrow \quad \overline{u'' u''} = 0.35k + \frac{2}{3} k$$

- Supressing of eddy viscosity

$$\mu_T = 0 \quad \Rightarrow \quad \overline{u'' u''} = \frac{2}{3} k$$



DNS data

Direct numerical simulation(DNS) of STI is presented by Mahesh *et al.* (1997)

M_1	M_t	Re_λ	k_{in}	ϵ_{in}
1.29	0.14	19.1	9.8×10^{-3}	1.3×10^{-3}
2.0	0.11	19.0	6.6×10^{-3}	6.0×10^{-3}
3.0	0.11	19.7	6.6×10^{-3}	5.7×10^{-3}

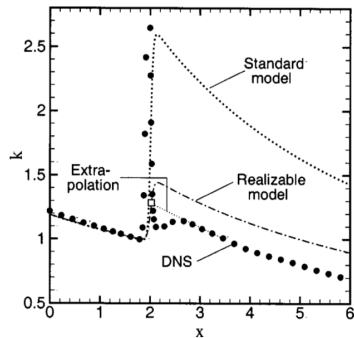
Table : Mean and turbulent flow quantities for the interaction of homogeneous turbulence with a normal shock

where $Re_\lambda = u_{rms}\lambda/\bar{\nu}$, and $M_t = \sqrt{2k}/\tilde{a}$,

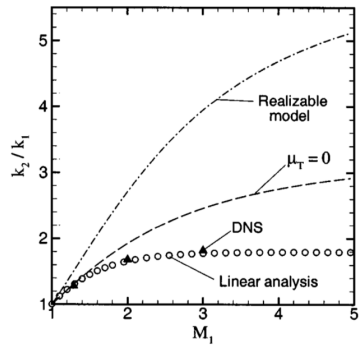
u_{rms} is the rms velocity, λ is the Taylor micro-scale, $\bar{\nu}$ is the mean kinematic viscosity of the fluid, $\tilde{a}$ is the mean speed of sound.



Amplification across the shock



(a)



(b)

Amplification and streamwise evolution of Turbulent kinetic energy



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Modeling for unsteadiness of shock (Sinha *et al.*, 2003)

$$\rho(u - \xi_t) \frac{\partial u}{\partial x} + \frac{\partial p}{\partial x} = 0$$

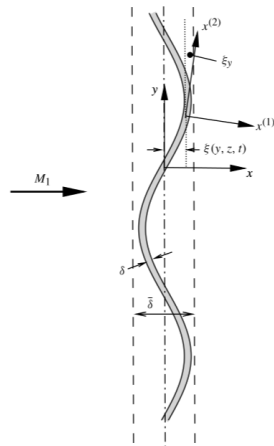
$$\widetilde{\rho} \widetilde{u} \frac{\partial \widetilde{u}''^2}{\partial x} \frac{1}{2} = \underbrace{-\widetilde{\rho} \widetilde{u}''^2 \frac{\partial \widetilde{u}}{\partial x}}_{P_k} + \underbrace{\widetilde{\rho} \widetilde{u}'' \xi_t \frac{\partial \widetilde{u}}{\partial x}}_{S_k^1} - \underbrace{\widetilde{u}'' \frac{\partial \bar{p}}{\partial x}}_{\Pi_k^1} - \underbrace{\widetilde{u}' \frac{\partial p'}{\partial x}}_{\Pi_k^2}$$

$$\widetilde{\rho} \widetilde{u} \frac{\partial \widetilde{v}''^2}{\partial x} \frac{1}{2} = - \underbrace{\widetilde{\rho} \widetilde{u} \widetilde{v}'' \xi_y \frac{\partial \widetilde{u}}{\partial x}}_{S_k^2}$$

$$u = \widetilde{u} + u'', p = \bar{p} + p'$$

$x = \xi(y, z, t)$, distortion of shock from its mean position;

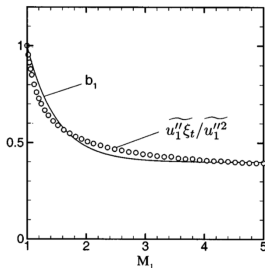
ξ_t, ξ_y, ξ_z are linear velocity of the shock, angular distortion in the $x - y$ and $x - z$ plane respectively.



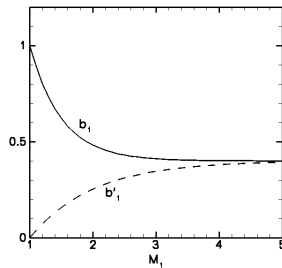
Using Linear Interaction analysis

$$\bar{\rho} \widetilde{u} \frac{\partial k}{\partial x} = P_k + \textcolor{red}{S}_k^1 + 2S_k^2 + \Pi_k^1 + \Pi_k^2$$

$$S_k^1 = \bar{\rho} \widetilde{u''^2} \frac{\partial \widetilde{u}}{\partial x} b_1 \quad \Rightarrow \quad b_1 = \widetilde{u'' \xi_t} / \widetilde{u''^2}$$



(a) $b_1 = 0.4 + 0.6e^{2(1-M_1)}$



(b) $b'_1 = b_{1,\infty} (1 - e^{(1-M_1)})$

Variation of $\widetilde{u'' \xi_t} / \widetilde{u''^2}$ as a function of upstream Mach number predicted by linear analysis, used to obtain model parameter b_1, b'_1



Turbulent transport equations

$$\begin{aligned}\bar{\rho}\tilde{u}\frac{\partial k}{\partial x} &= -\frac{2}{3}\bar{\rho}k\frac{\partial\tilde{u}}{\partial x}(1 - b'_1) - \bar{\rho}\epsilon_s \\ \bar{\rho}\tilde{u}\frac{\partial\epsilon_s}{\partial x} &= -\frac{2}{3}C_{\epsilon 1}\bar{\rho}\frac{\partial\tilde{u}}{\partial x}\epsilon_s - C_{\epsilon 2}\frac{\epsilon_s^2}{k}\end{aligned}$$

Analytical integrating across the shock,

$$\frac{k_2}{k_1} = \left[\frac{\tilde{u}_1}{\tilde{u}_2} \right]^{\frac{2}{3}(1-b'_1)}, \quad \frac{\epsilon_{s2}}{\epsilon_{s1}} = \left[\frac{\tilde{u}_1}{\tilde{u}_2} \right]^{\frac{2}{3}C_{\epsilon 1}}$$

where $C_{\epsilon 1} = 1.25 + 0.2(M_1 - 1)$ and $C_{\epsilon 2} = 1.8$.

Note

Across the shock, turbulent dissipation rate has a negligible effect on the evolution of k and ϵ_s .

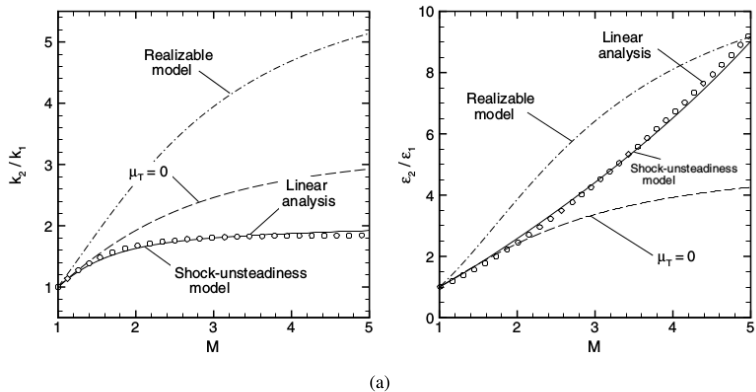


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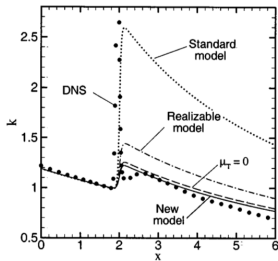
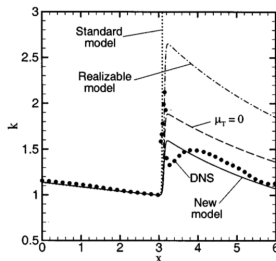
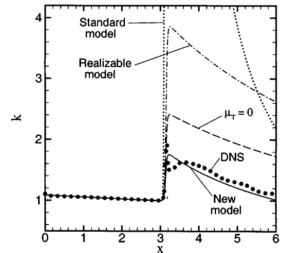
Amplification across the shock



Amplification of Turbulent kinetic energy and solenoidal dissipation rate in a shock/turbulence interaction as a function of the upstream Mach number



Streamwise evolution of turbulent kinetic energy

(a) $M=1.29$ (b) $M=2.0$ (c) $M=3.0$

Evolution of k in the interaction of homogeneous isotropic turbulence with a normal shock.



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Summary

- The standard $k - \epsilon$ model grossly over-predicts the amplification in the turbulent kinetic energy, k , across the shock, because the underlying eddy viscosity assumption breaks down in a rapidly distorting mean flow.
- Modifications based on the realizability constraint reduce the eddy viscosity, and thus yield a lower amplification in k .
- The equation for the solenoidal dissipation rate is also modified so that it predicts the correct amplification of ϵ across the shock.
- The new $k - \epsilon$ reproduces DNS data of shock/isotropic turbulence interaction well.



Thank you...