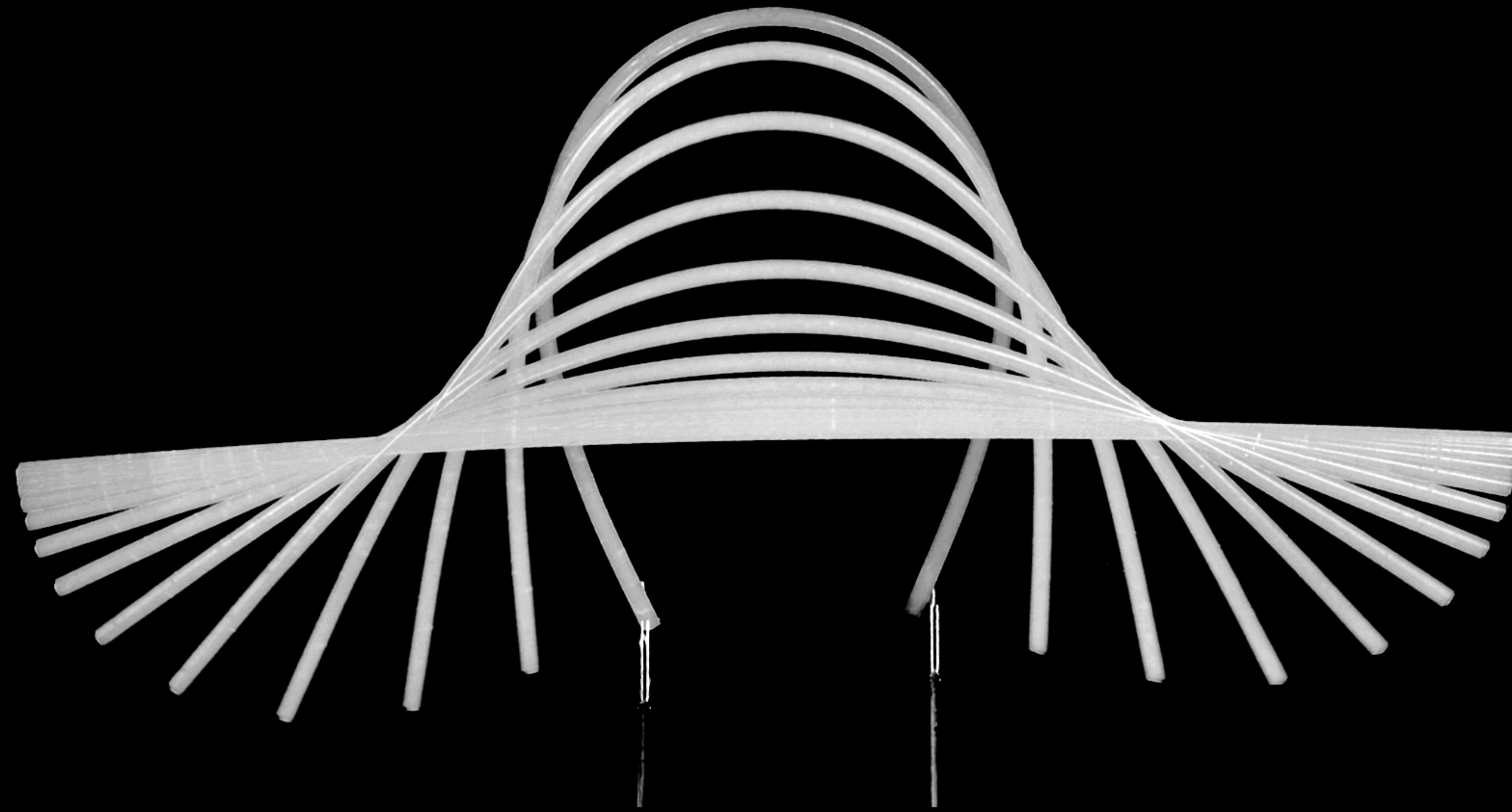


Large deformation dynamics of an elastic filament



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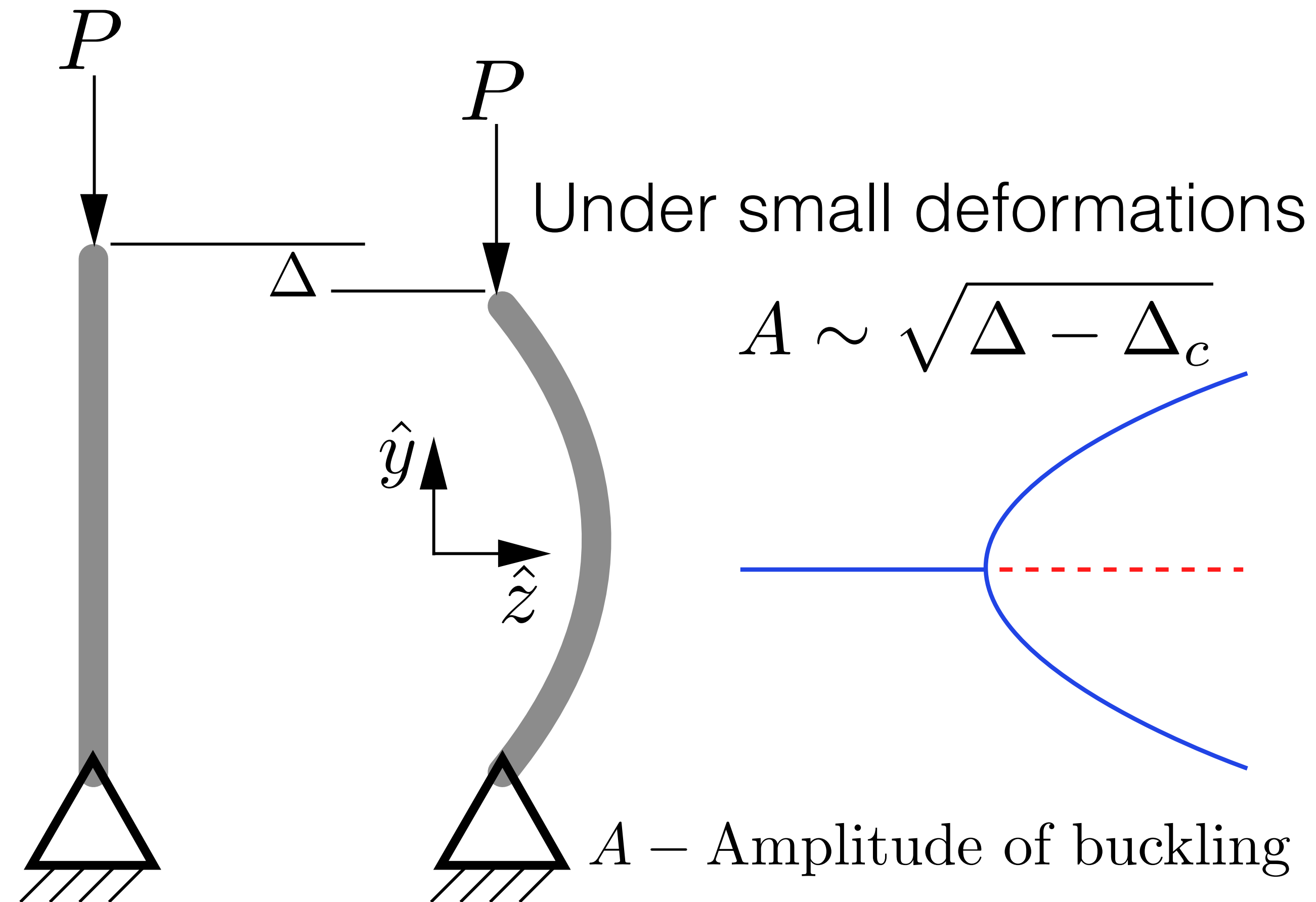


1D elastic structures

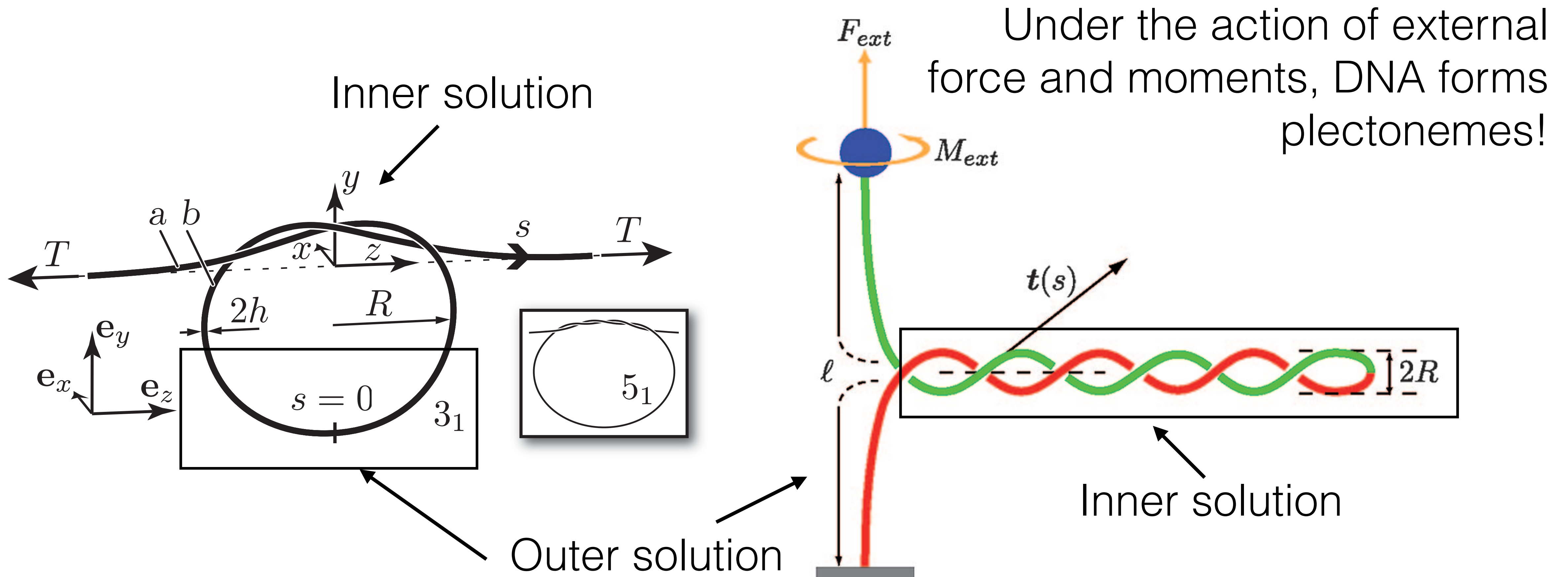
Filaments in nature

Euler buckling

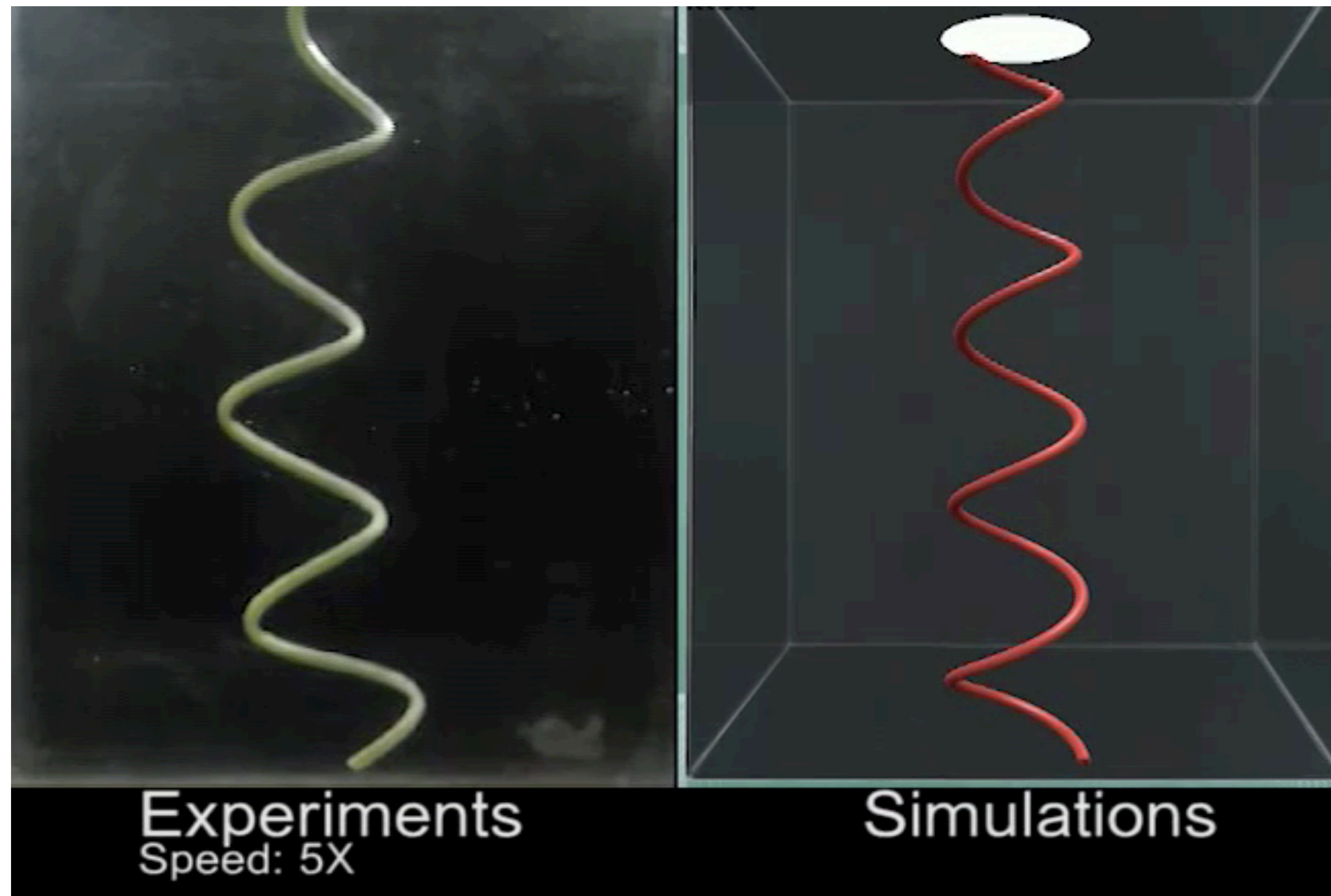
- Filaments undergo a variety of instabilities
- Depend on type of applied force
- And material properties of filament
- Simplest of them is buckling instability of Euler
- Filament with finite stretching & bending modulus undergoes pitchfork bifurcation



Knots & DNA plectonemics



Instability of flexible helical rod

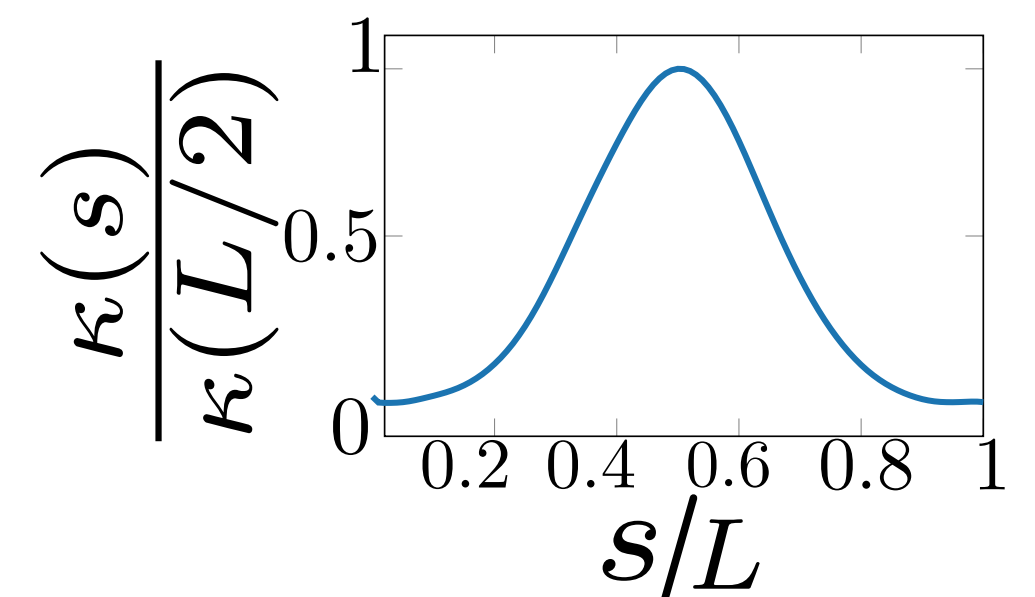
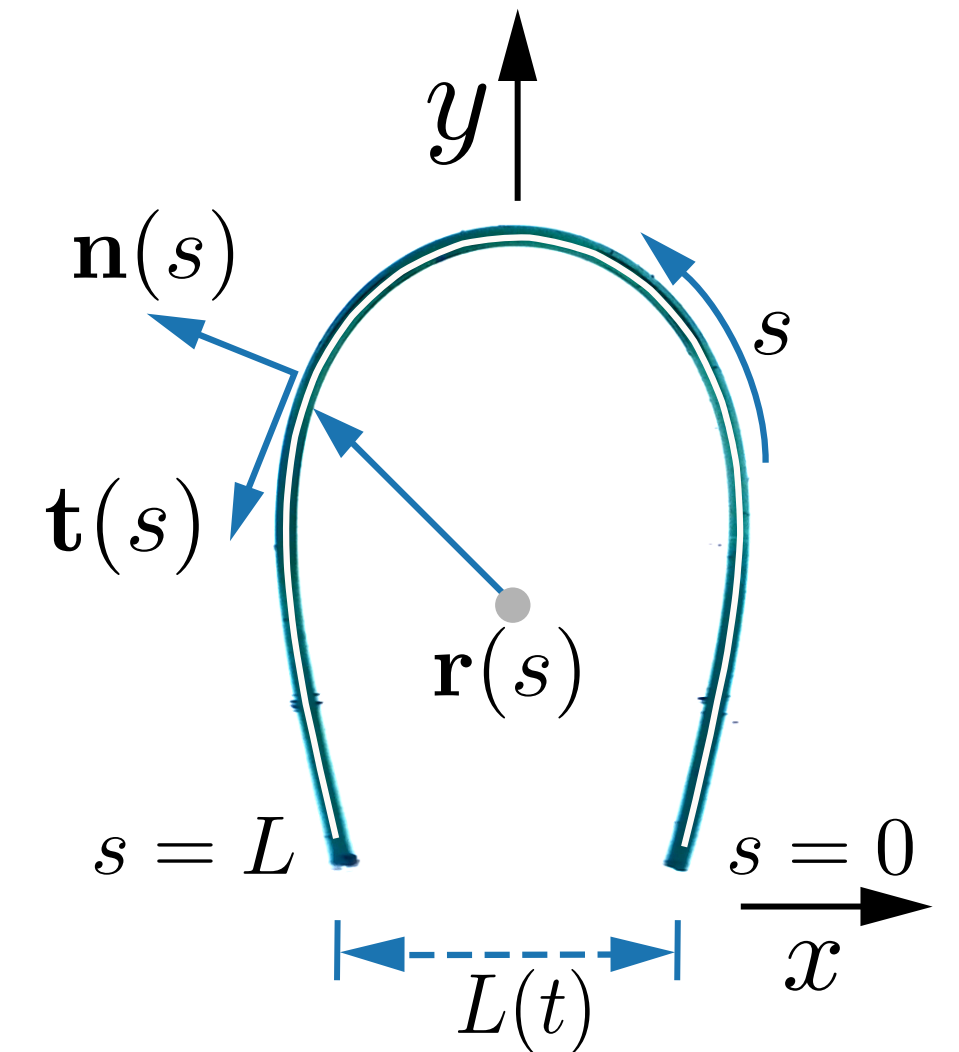


Experiments

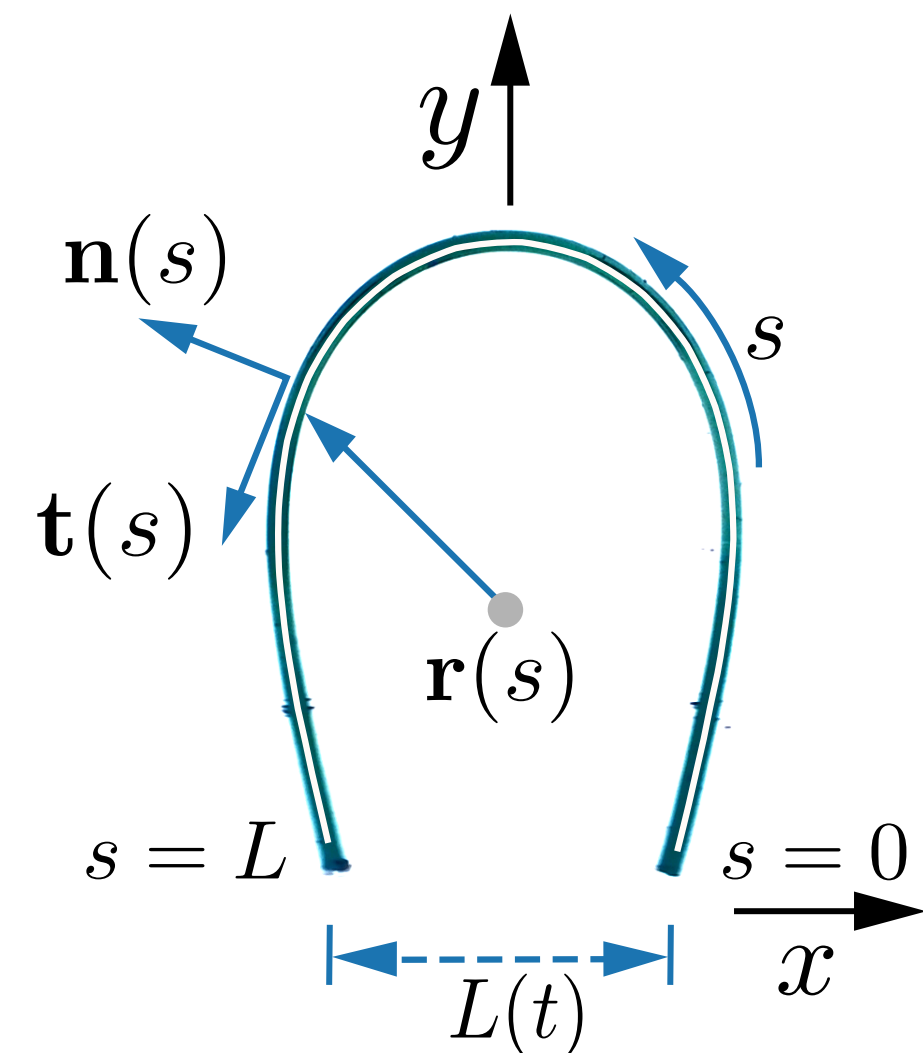
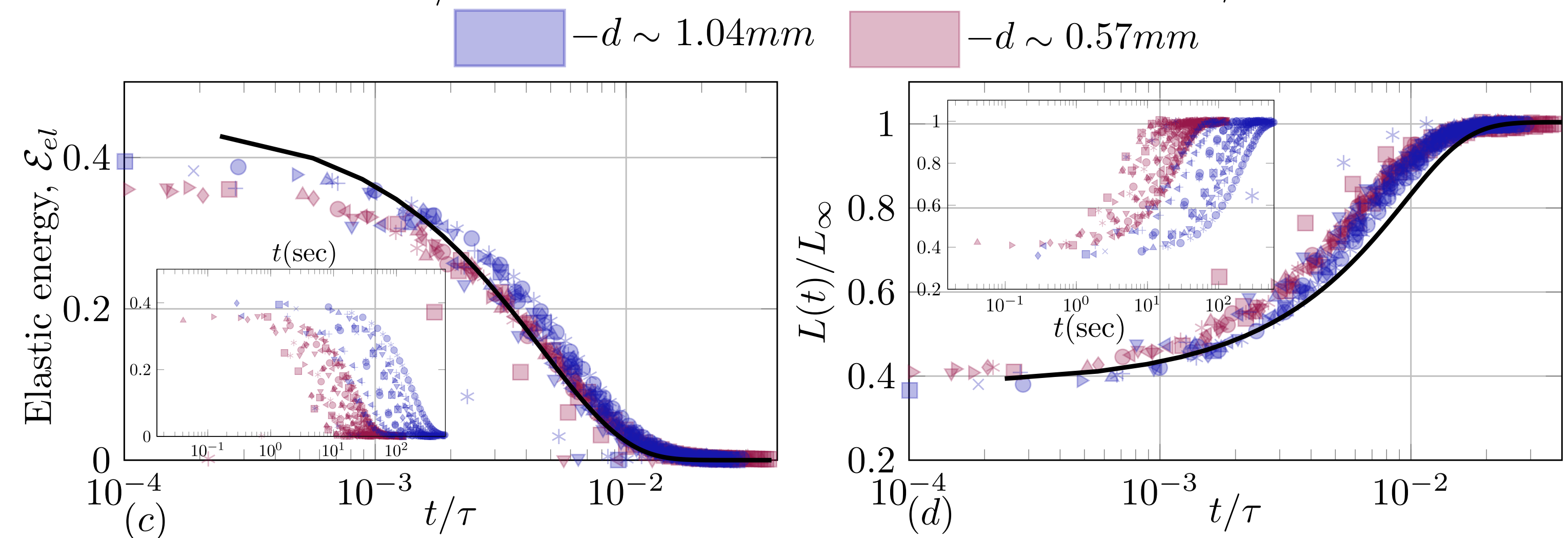
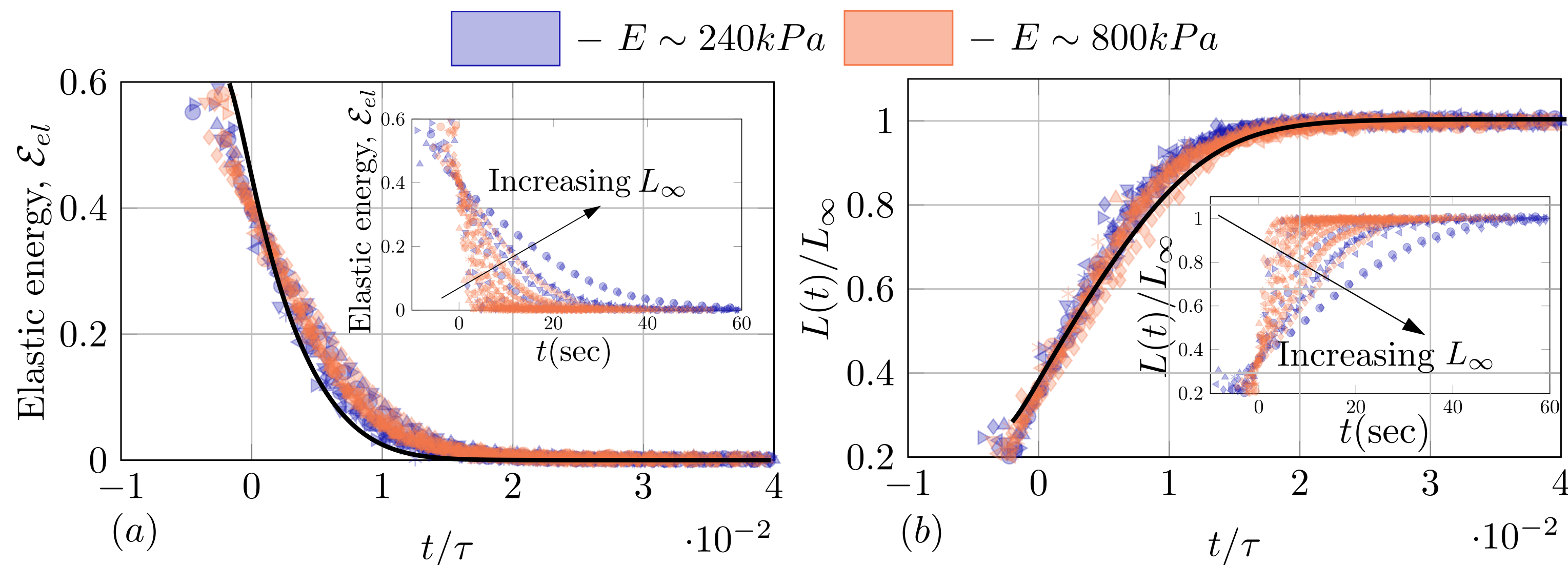


E ($800kPa, 240kPa$)
 $d = 0.57mm, 1.04mm$
 $L_{\infty} 4.6cm - 7.2cm$

Glycerol as
working fluid



Experiments



$$\mathcal{E}_{el} = \frac{B}{2} \int_0^{L_\infty} \kappa^2(s) ds$$

B — Bending modulus

$$\tau = \frac{8\pi\mu L_\infty^4}{B}$$

$$\mathcal{E}_{el} \sim d^4, \tau \sim d^{-4}$$

Overall factor of 26.6
between slowest and fastest
time of relaxation

Theoretical model

$$\mathcal{E}_{el} = \frac{B}{2} \int_0^{L_\infty} \kappa^2(s) ds + \int_0^{L_\infty} \frac{T(s)}{2} \{ |\mathbf{t}(s)|^2 - 1 \} ds$$

↑
Elastic energy
↑
Length constraint

- Isotropic drag due to fluid
- Hydrodynamic interactions neglected
- Wetting properties neglected

$$8\pi\mu\partial_t\mathbf{r} = -B\mathbf{r}_{ssss} + \partial_s[T(s)\mathbf{r}_s]$$

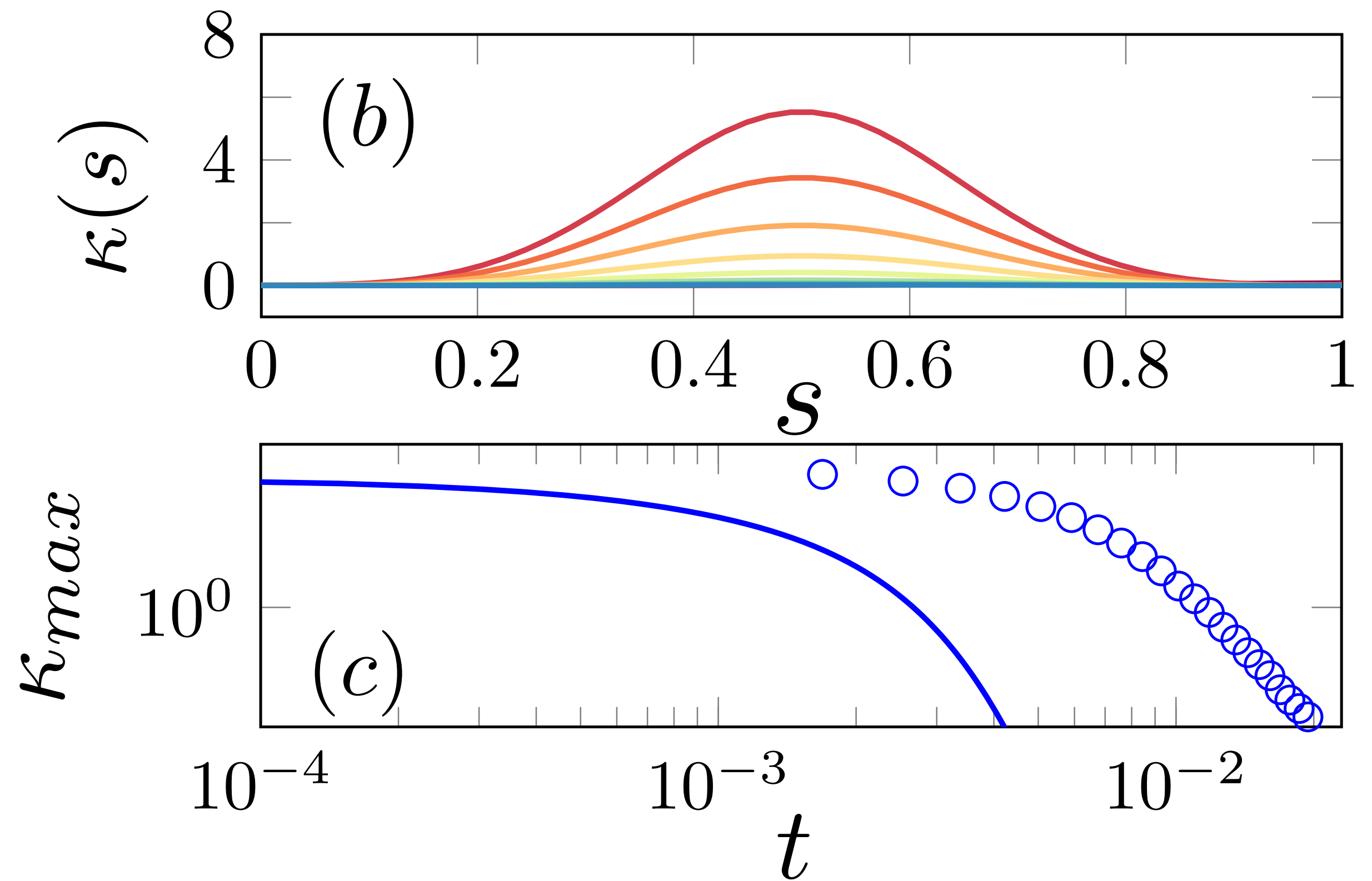
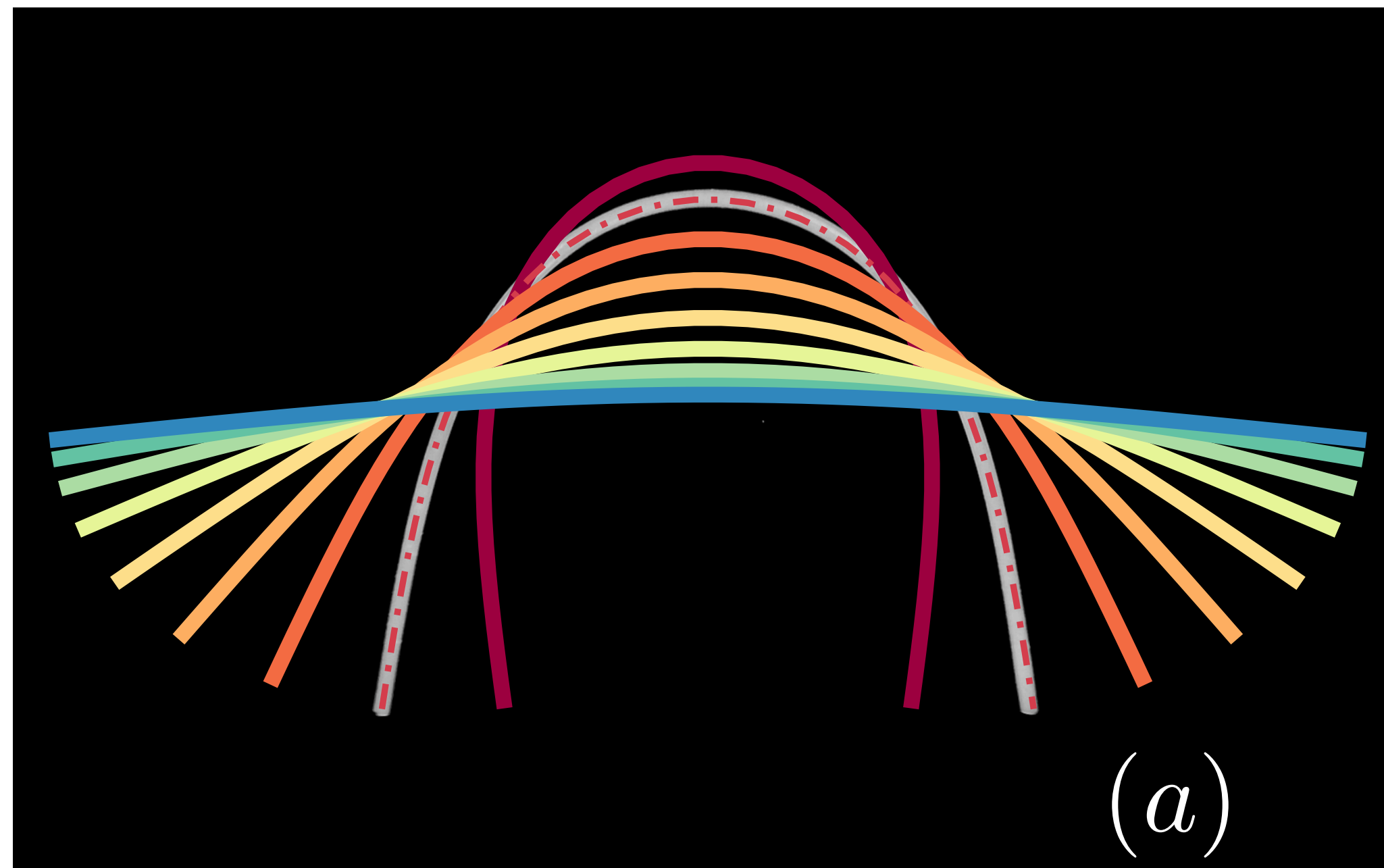
$$(\partial_{ss} - |\mathbf{r}_{ss}|^2)T(s) = -(3|\mathbf{r}_{sss}|^2 + 4(\mathbf{r}_{ss} \cdot \mathbf{r}_{ssss}))$$

Boundary conditions

$$\mathbf{r}_{ss}(0) = \mathbf{r}_{sss}(0) = \mathbf{r}_{ss}(L_\infty) = \mathbf{r}_{sss}(L_\infty) = 0$$

Zero moment and force

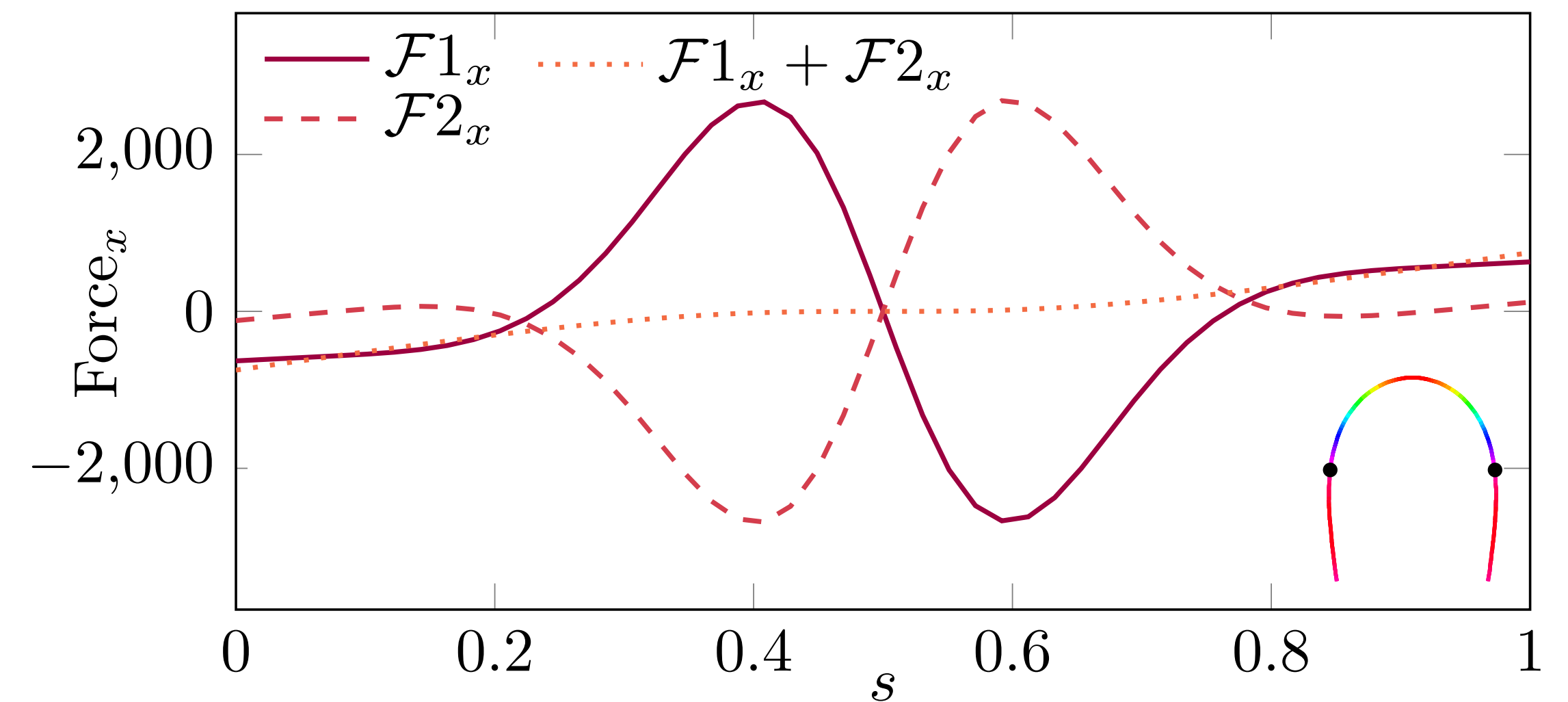
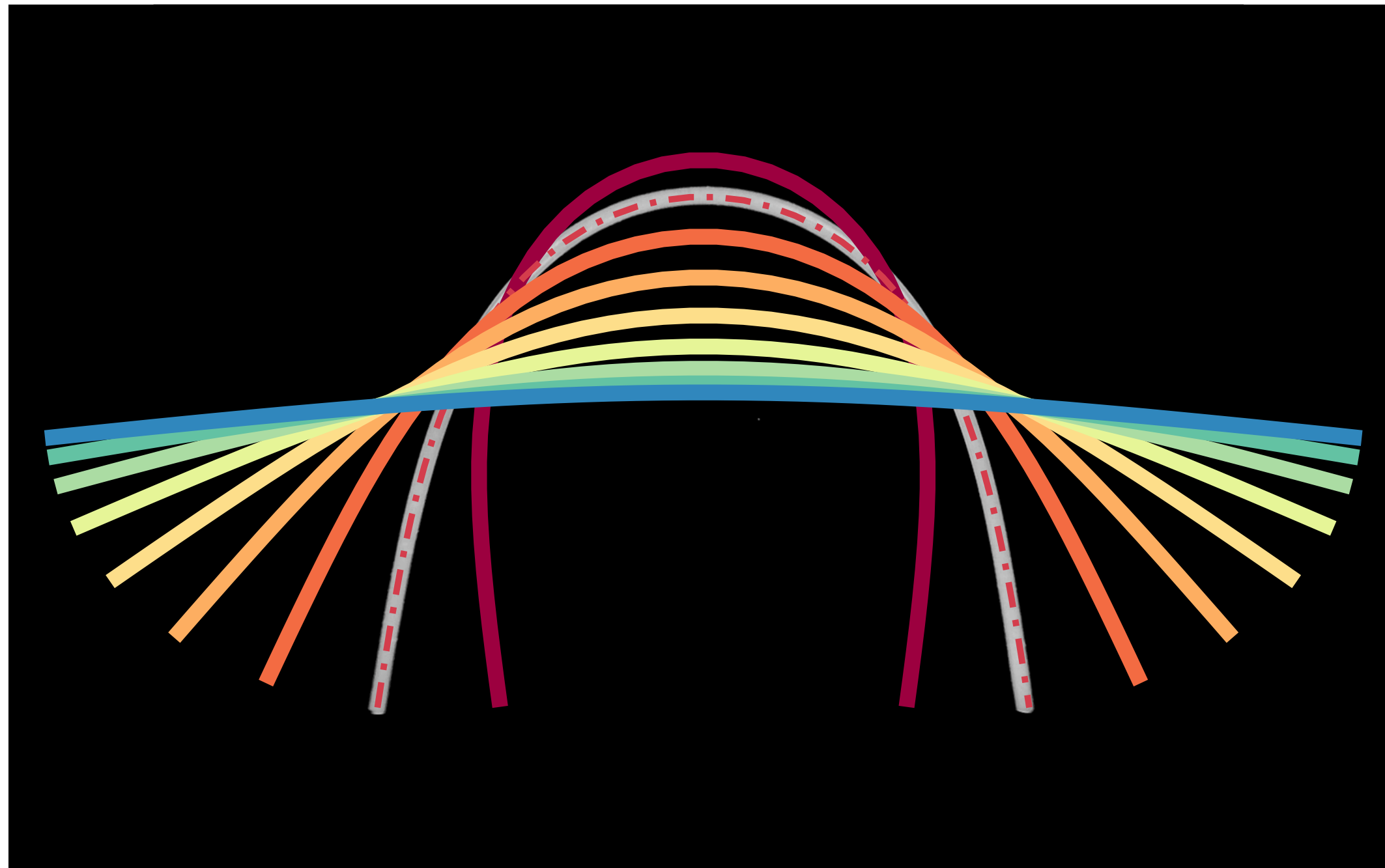
Numerics



Simulation captures experiments except for an $O(1)$ number!

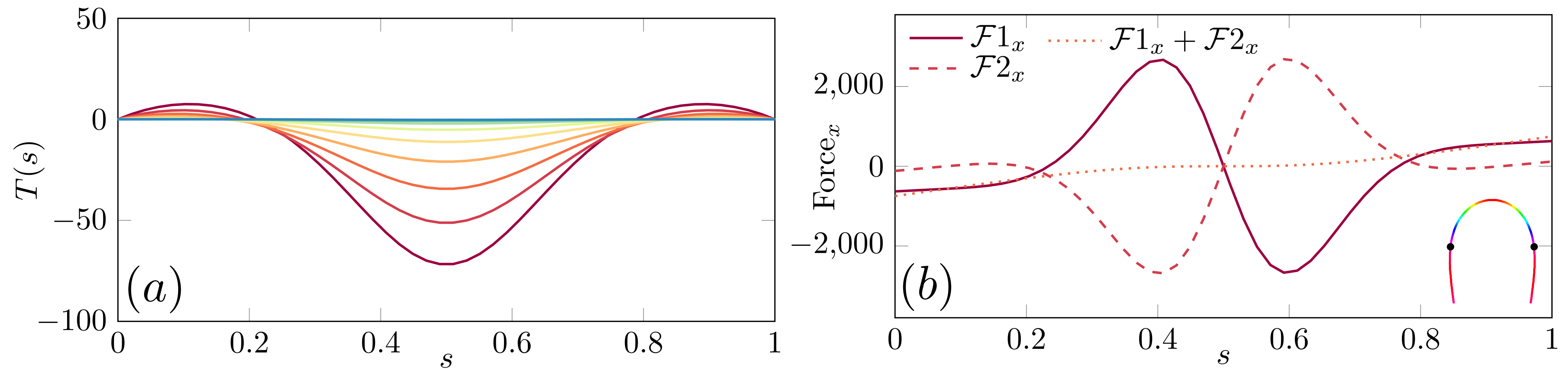
Results

$$\bar{\mu} \partial_t \mathbf{r} = - \underbrace{\mathbf{r}_{ssss}}_{\mathcal{F}_1} + \underbrace{\partial_s [T(s) \mathbf{r}_s]}_{\mathcal{F}_2}$$



Force due to tension and bending
very similar in magnitude

Tension



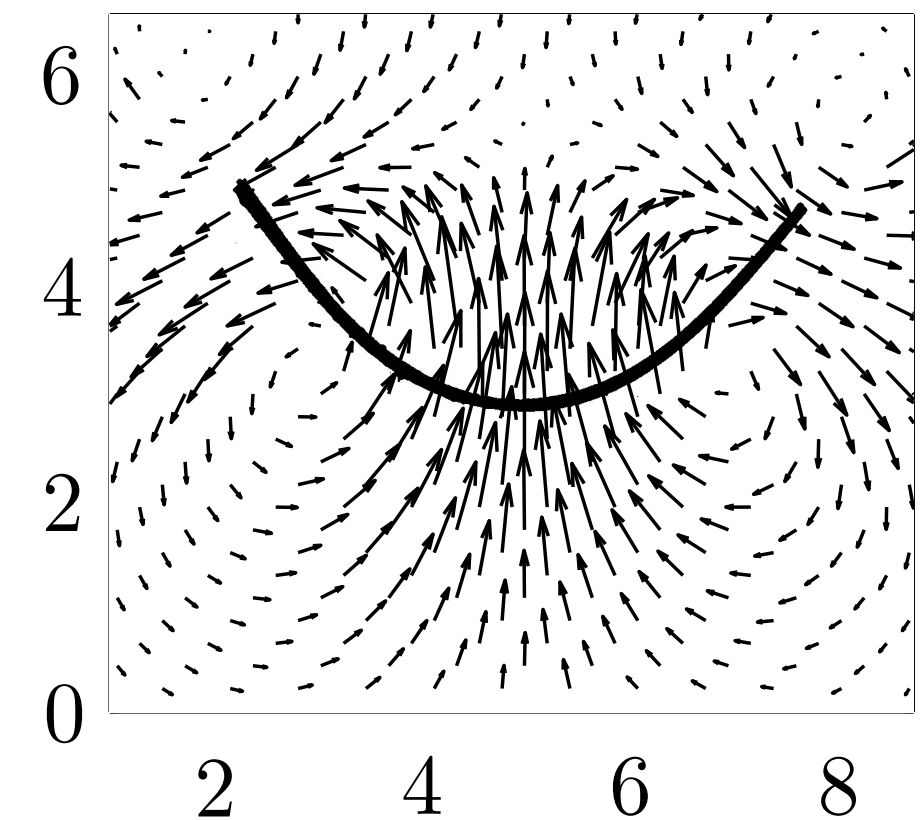
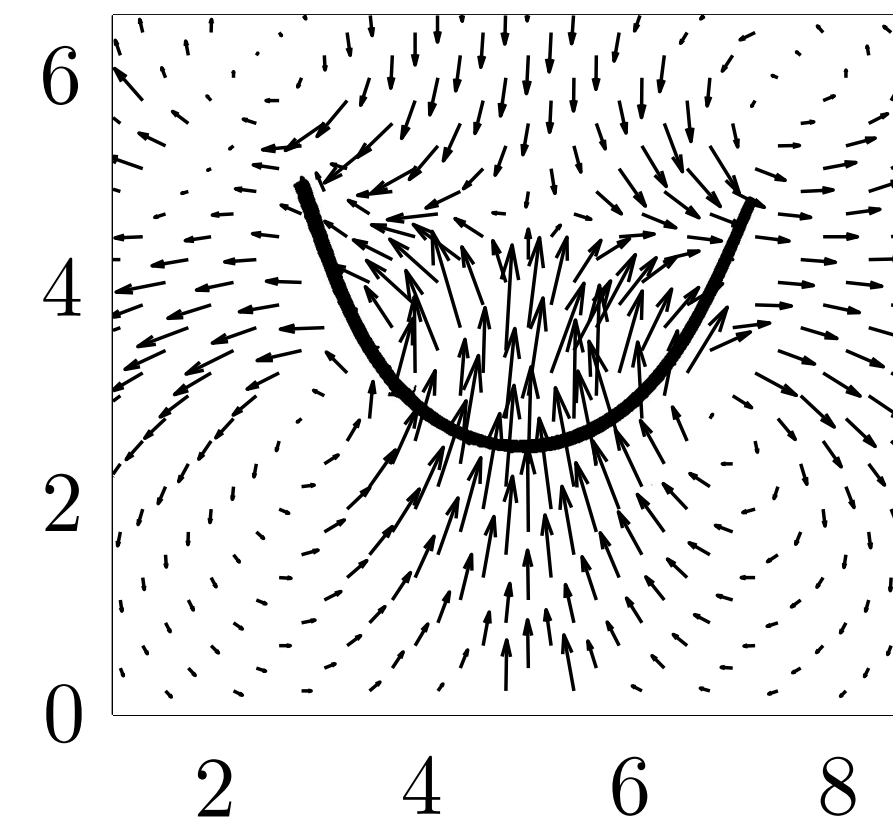
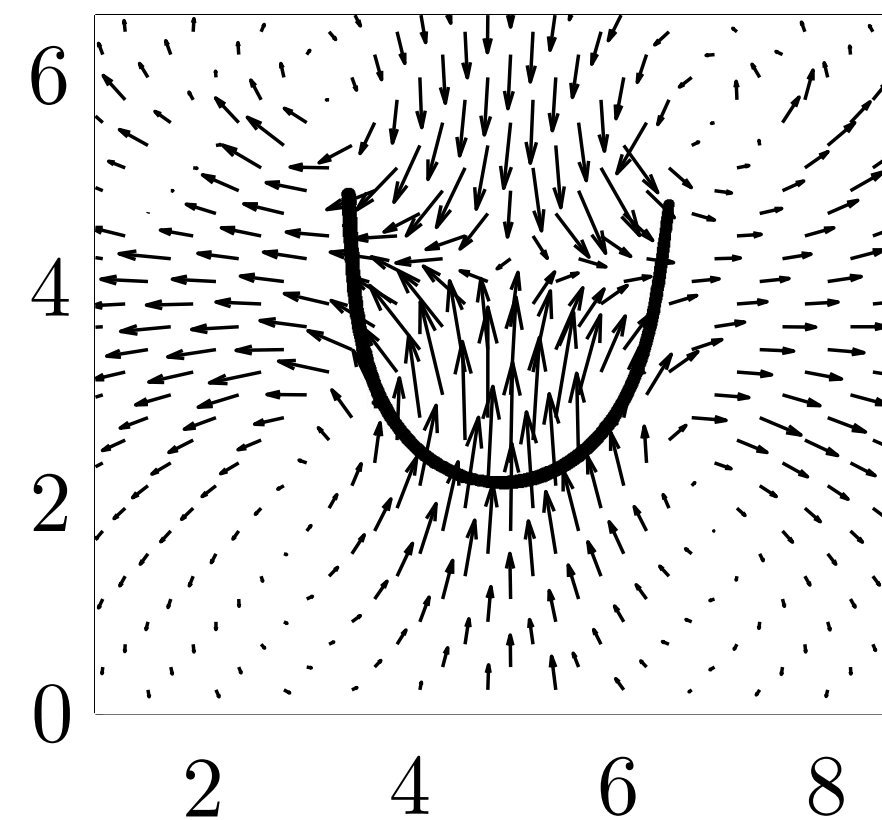
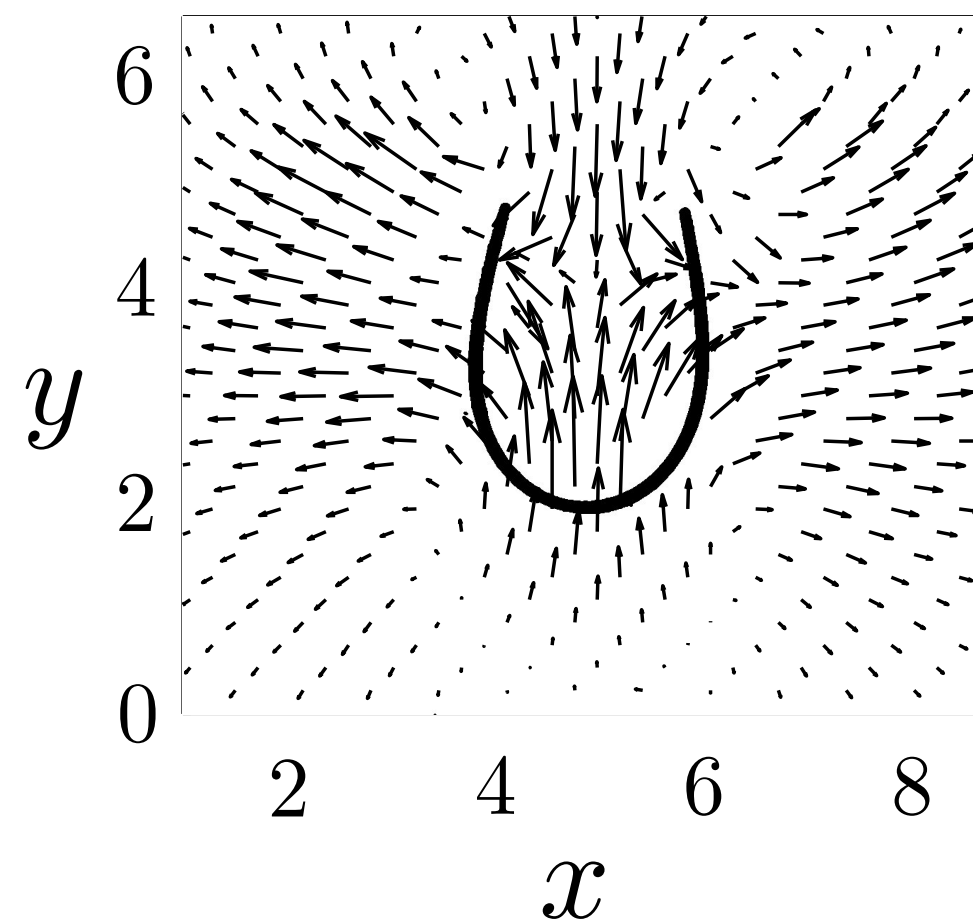
- Tension is inevitable at large deformations
- Without this, viscous dissipation over-predicted

Tension

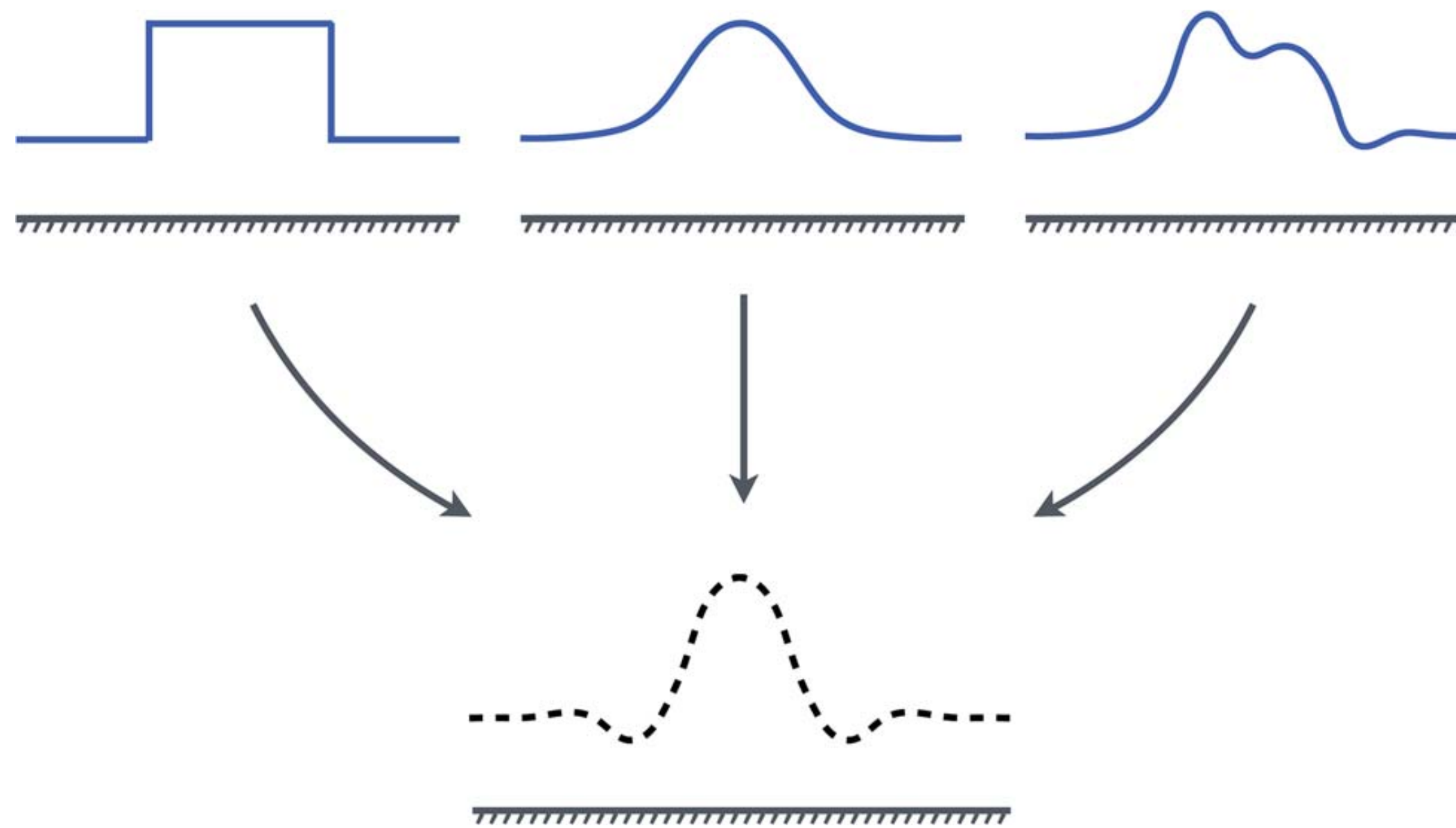
Location where Tension goes to zero depends on:

$$\mathbf{r}_s \cdot \hat{\mathbf{t}} \partial_s T(s) = \bar{\mu} \partial_t \mathbf{r} \cdot \hat{\mathbf{t}} + \mathbf{r}_{ssss} \cdot \hat{\mathbf{t}}$$

$$T(s) = \int_0^s (\mathbf{r}_{ssss} \cdot \hat{\mathbf{t}} + \bar{\mu} \mathbf{r}_t \cdot \hat{\mathbf{t}}) ds$$



Case of thin-films & infinitely long filaments



Happens to be the same equation for filaments in the linear limit

$$\bar{\mu}\Omega = -\iota k^4$$

$$\partial_t h + \frac{\gamma}{3\eta} \partial_x (h^3 \partial_x^3 h) = 0$$

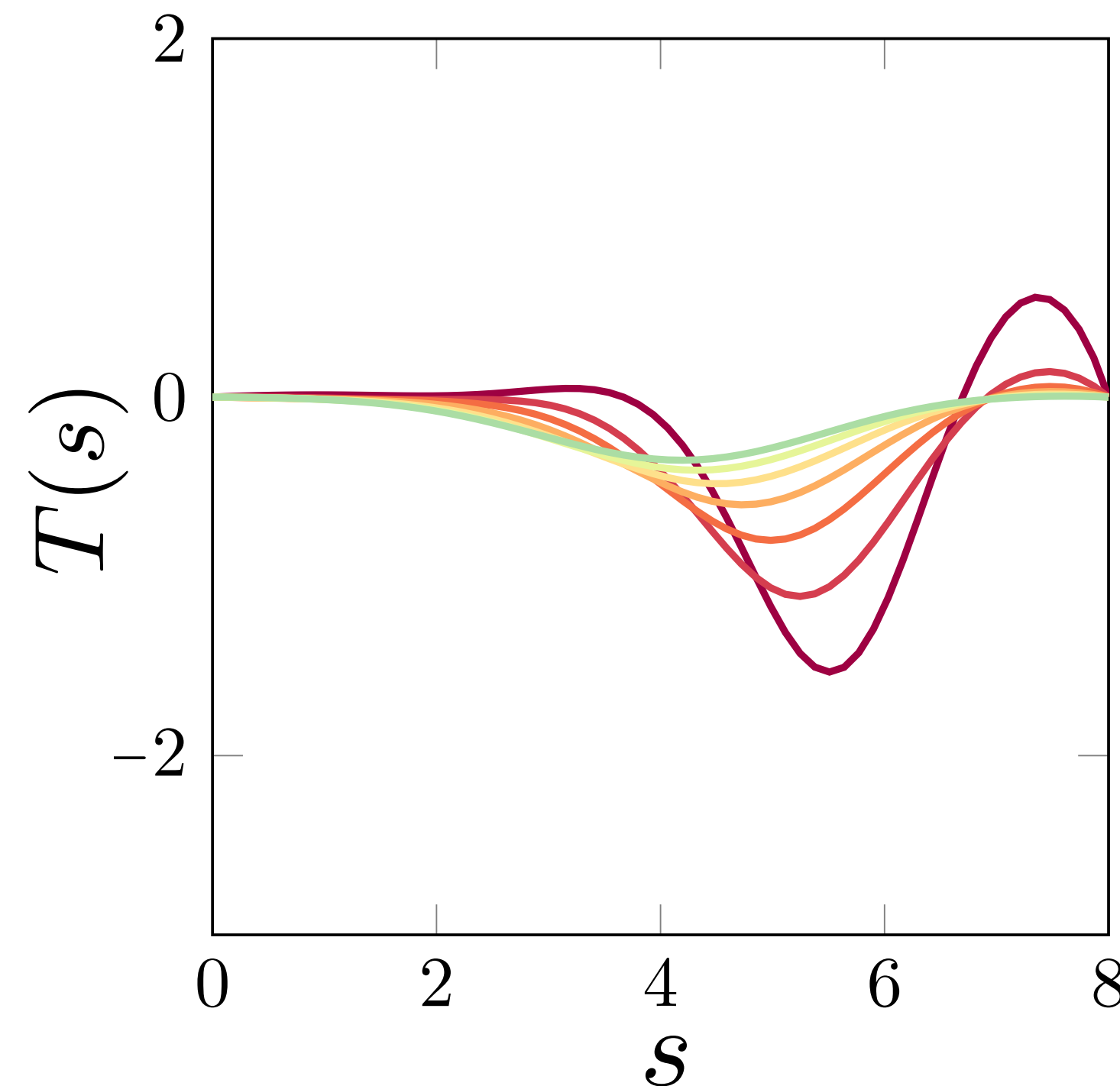
$$\partial_T \Delta + \partial_x^4 \Delta = 0$$

Asymmetric initial condition

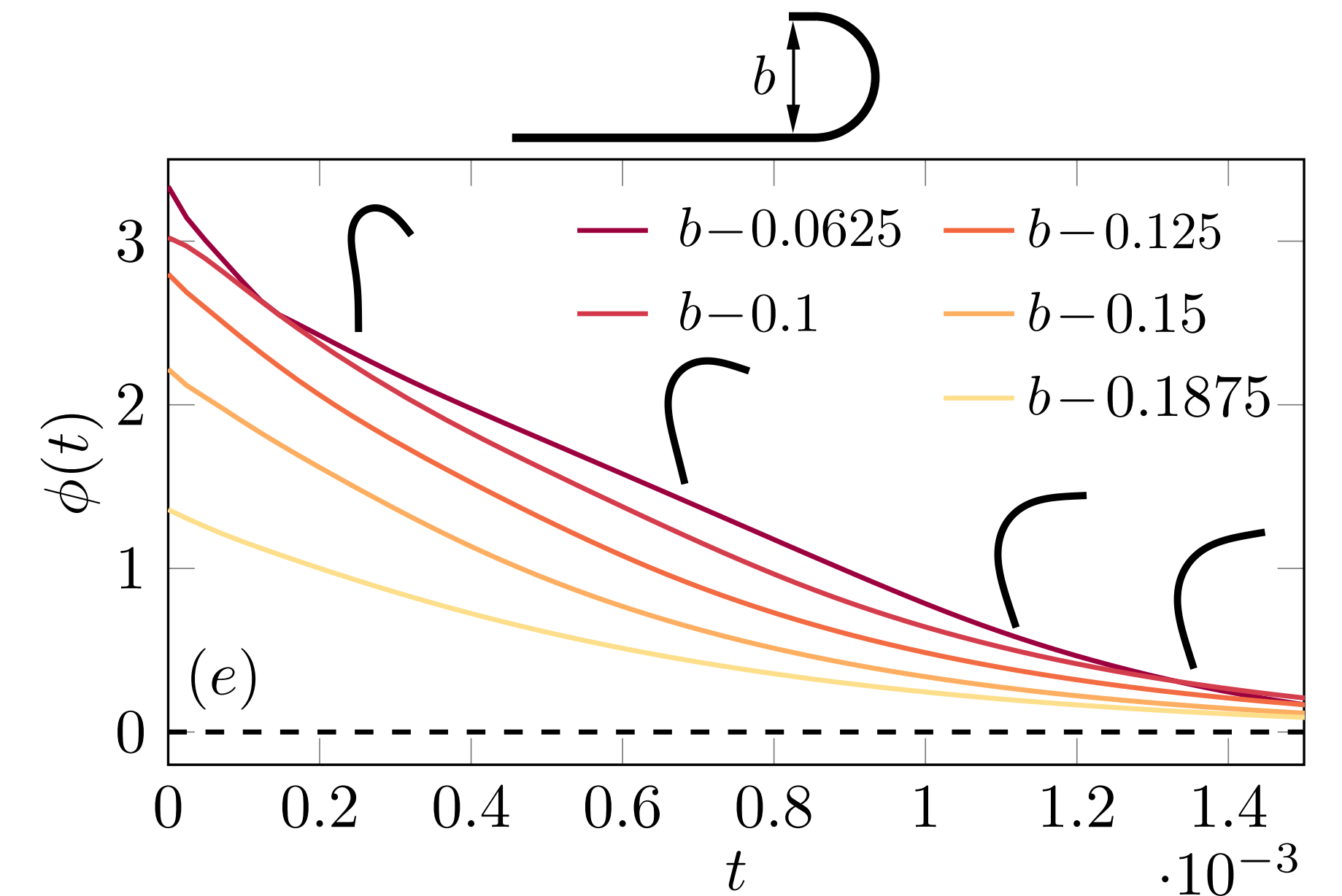


- Filaments that start with asymmetric initial condition move to symmetric stresses
- After attaining symmetric stress, they continue to remain symmetric
- This seems to be the case for any arbitrary initial asymmetric perturbation (in expts.)

Asymmetric initial condition



$$\phi = \int_0^{L/2} |\kappa(L/2 - s) - \kappa(s)| ds$$



Tension is indispensable for this to happen

Asymmetric initial condition



Symmetric curvature minimizes total elastic energy

$$\begin{aligned}\mathcal{E}_{el} &= \frac{B}{2} \left[\int_{-L/2}^0 (\hat{\kappa}(s) - \epsilon(s))^2 ds + \int_0^{L/2} (\hat{\kappa}(s) + \epsilon(-s))^2 ds \right] \\ &= \frac{B}{2} \left[\int_{-L/2}^{L/2} \hat{\kappa}^2(s) ds + \int_0^{L/2} (\epsilon^2(-s) + \epsilon^2(s)) ds \right] \\ &\geq \frac{B}{2} \int_{-L/2}^{L/2} \hat{\kappa}^2(s) ds\end{aligned}$$

Tension is indispensable for this to happen

Thank you!