

Effect of transport property variations on high-speed flow stability

5th Inter Group Meeting, 2017

Bijaylakshmi Saikia

Under the guidance of

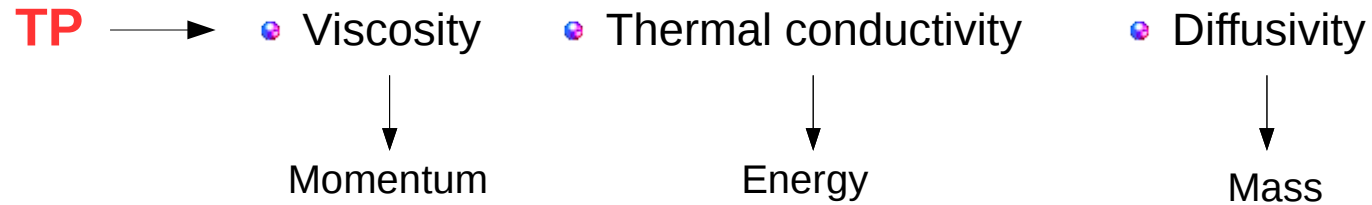
Prof. Krishnendu Sinha



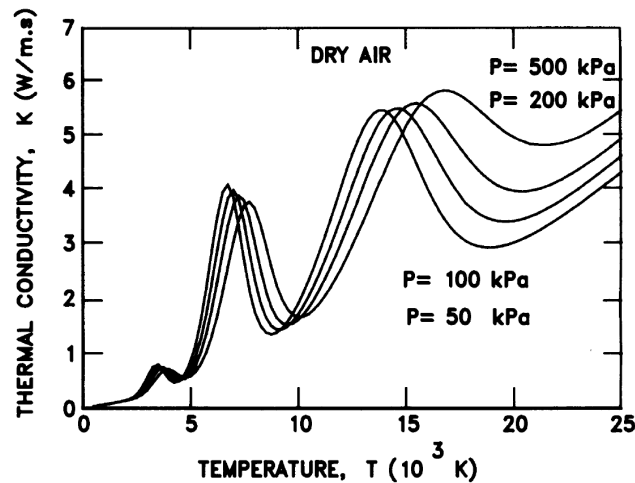
Department of Aerospace Engineering

Indian Institute of Technology Bombay, Mumbai

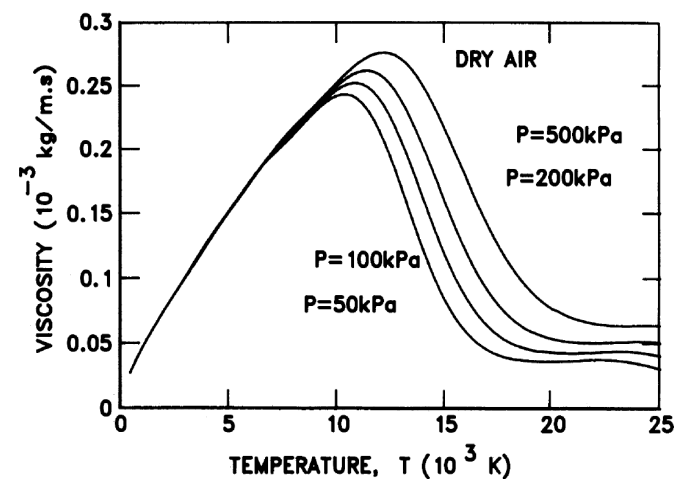
Transport properties in a high-speed flow: Large independent variations



- Large variation due to high-temperature effects
 - Internal mode excitation
 - Thermo-chemical non-equilibrium
- Significant effect on flow transition



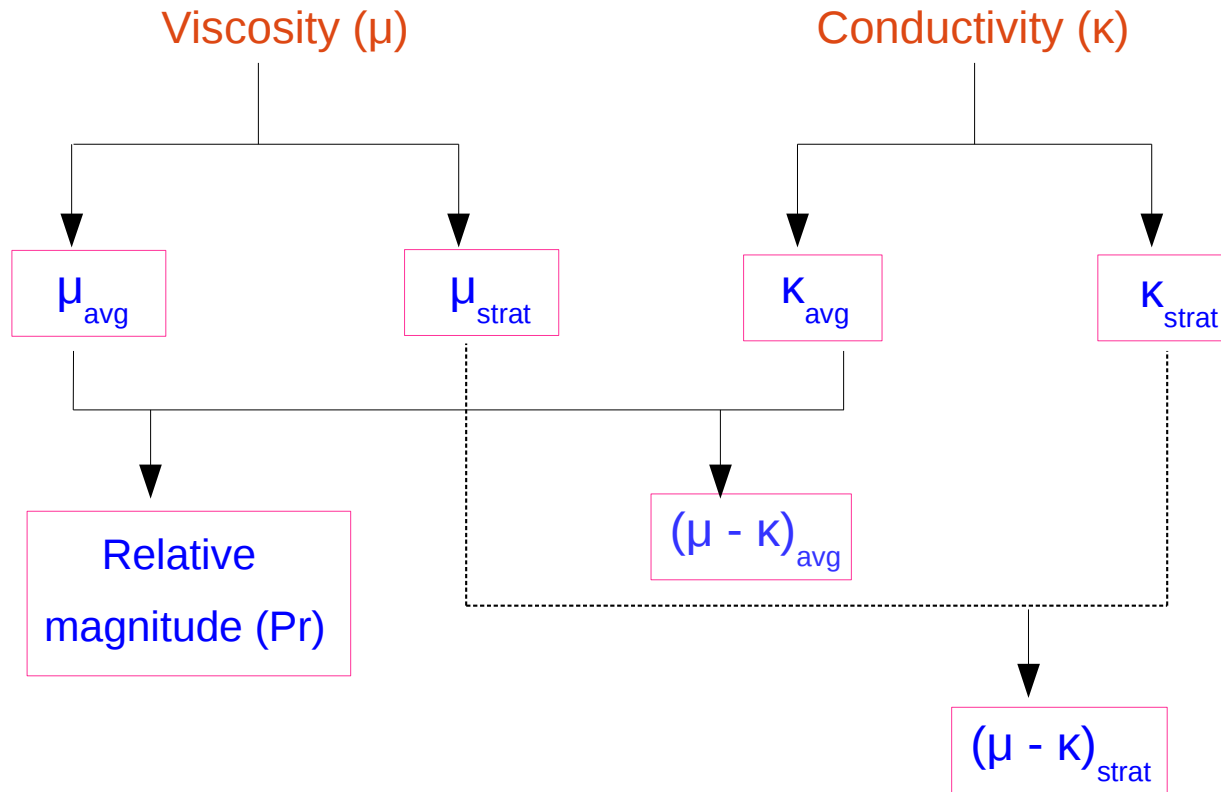
Conductivity



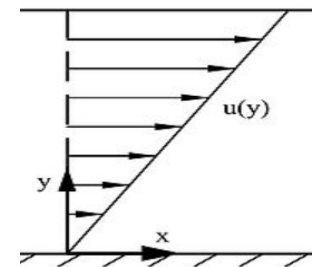
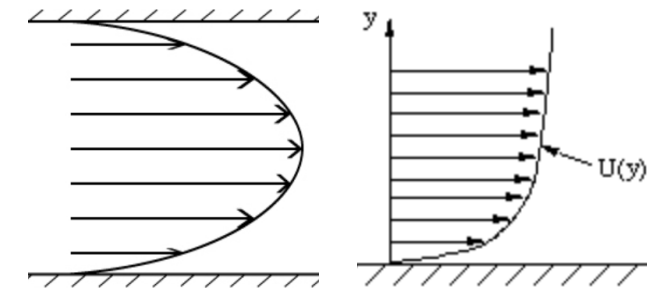
Viscosity

Objective

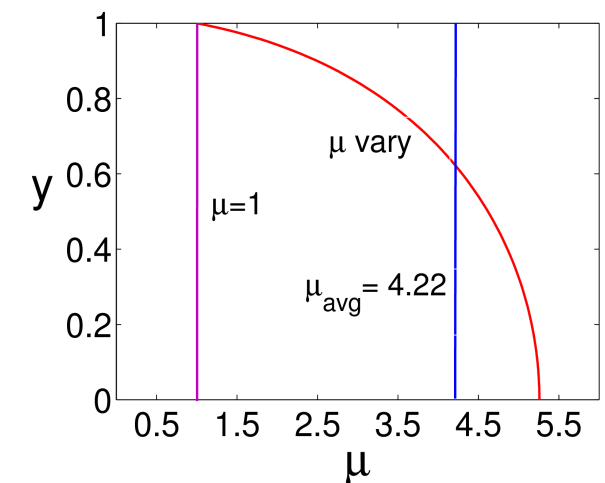
- Effect of the mean value + slope in the transport properties



- Reference flow
- Averaged flow
- Stratified flow

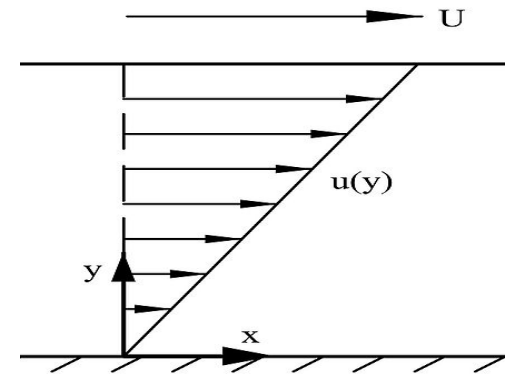


Shear flows



Model problem & stability approach

- Upper plate is moving, lower plate is at rest
- Perfect gas assumption
- Flow is steady & uni-directional
- Top wall variables are the reference quantities



Plane Couette flow

Governing equations:

X-momentum:

$$\frac{\partial}{\partial y}(\bar{\mu} \frac{\partial \bar{U}}{\partial y}) = 0$$

Energy equation:

$$\frac{\partial}{\partial y}(\bar{\kappa} \frac{\partial \bar{T}}{\partial y}) + (\gamma - 1) M^2 Pr \bar{\mu} (\frac{\partial \bar{U}}{\partial y})^2 = 0$$

- Linear stability theory (Temporal)

$$\tilde{A}(x, y, z, t) = \hat{A}(y) e^{\iota(\alpha x + \beta z - \omega t)}$$

$$\omega = \omega_r + \iota \omega_i$$

Disturbance grows if, $\omega_i > 0$

- Transient growth

$$G(t) = \max_{q_0} \frac{\|q(t)\|^2}{\|q(0)\|^2} = \|exp(tL)\|^2$$

Stability analysis of conductivity stratified flow

Base flow for conductivity stratified flow

X-momentum:

$$\bar{U} = y$$

Energy equation:

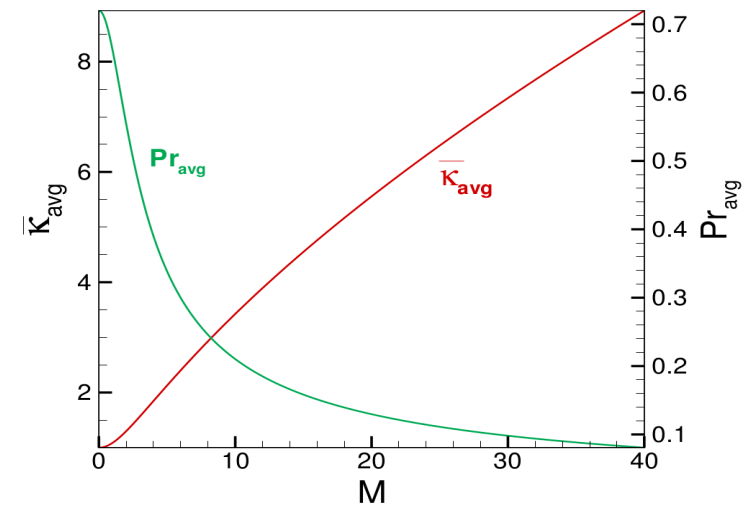
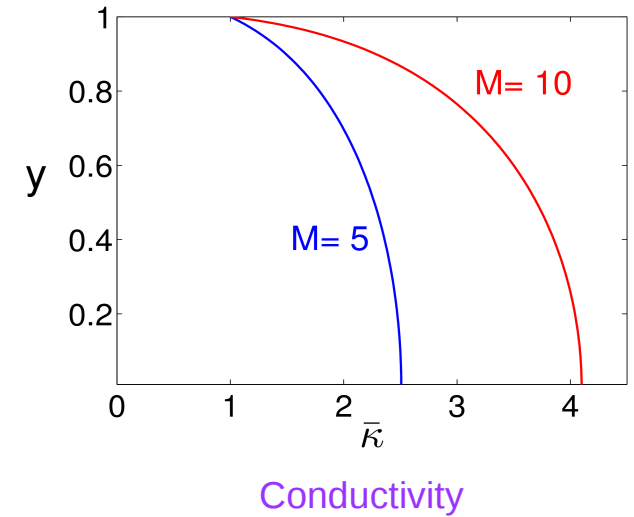
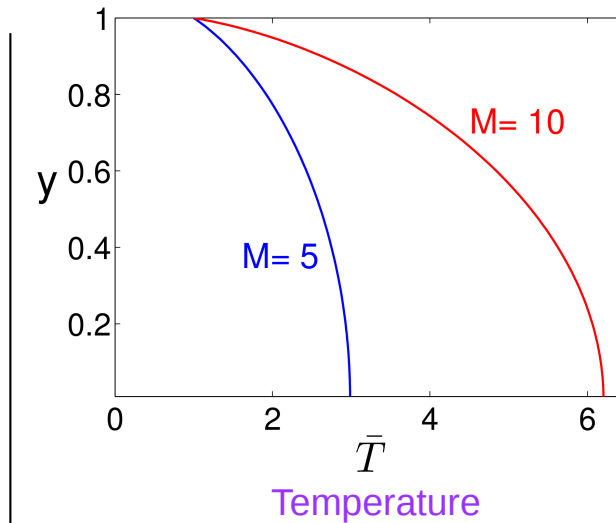
$$\frac{d\bar{T}}{dy} = -(\gamma - 1) M^2 Pr \frac{(\bar{T} + B)}{\bar{T}^{3/2} (1 + B)} y$$

Solved using RK -4

- κ varied using Sutherland's law

Average conductivity: $\bar{\kappa}_{avg} = \int_0^1 \bar{\kappa} dy$

Average Prandtl number: $Pr_{avg} = \frac{Pr}{\bar{\kappa}_{avg}}$



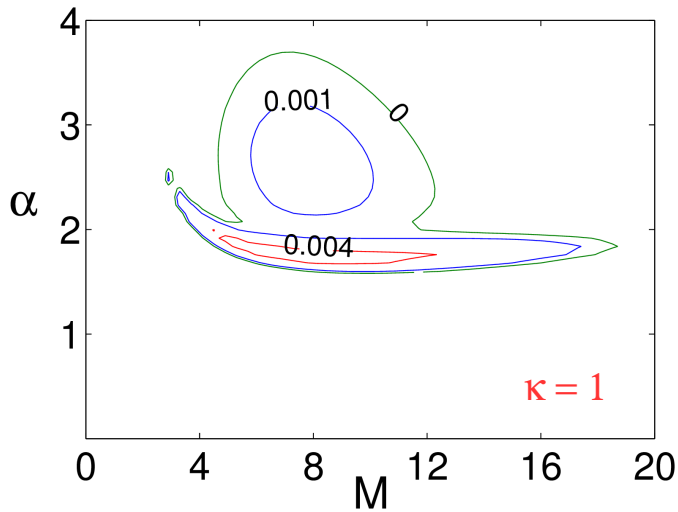
Average conductivity and Pr

$$Pr_{ref} = 0.72$$

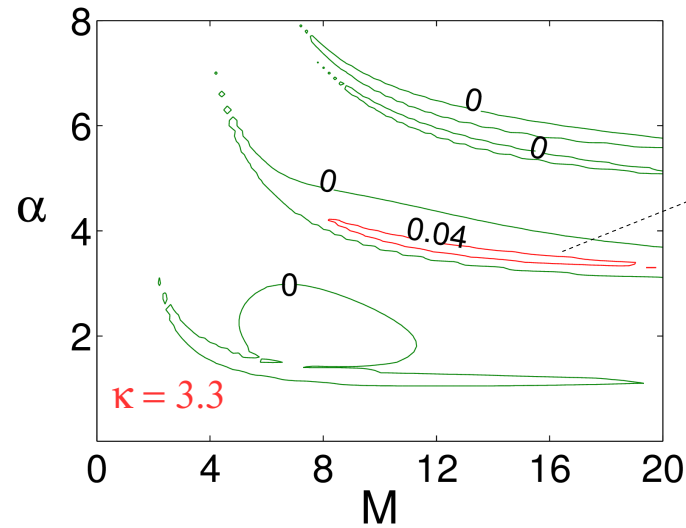
Effect of conductivity : LST prediction

$Re = 10^5$, $Pr = 0.72$

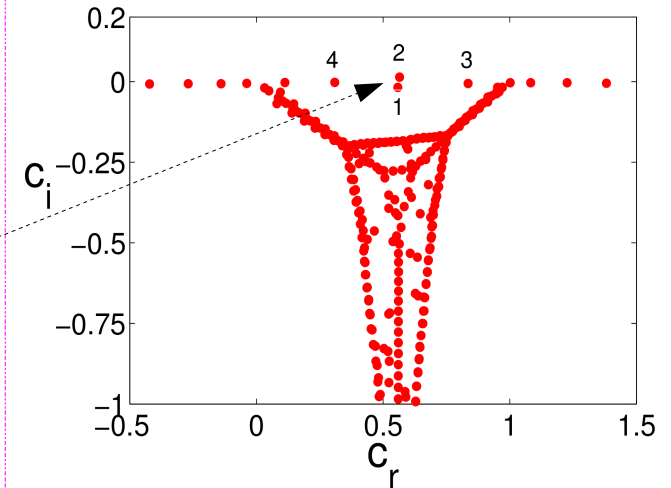
Reference flow



κ -averaged flow



Eigen spectrum



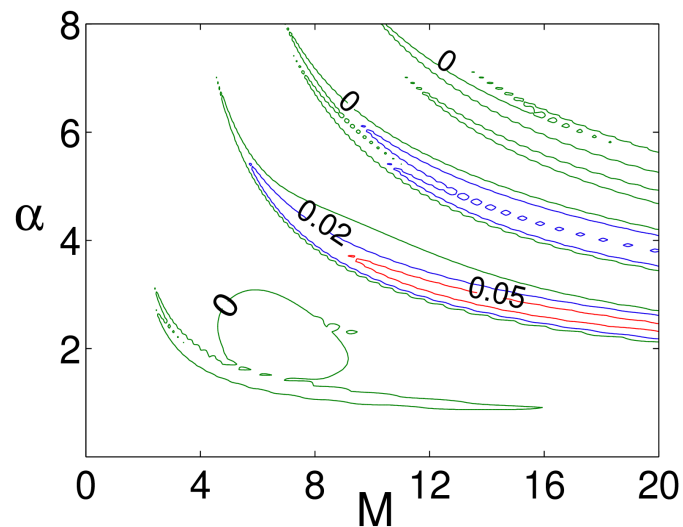
Increase in conductivity,

- Increases maximum growth rate
- More instability regions

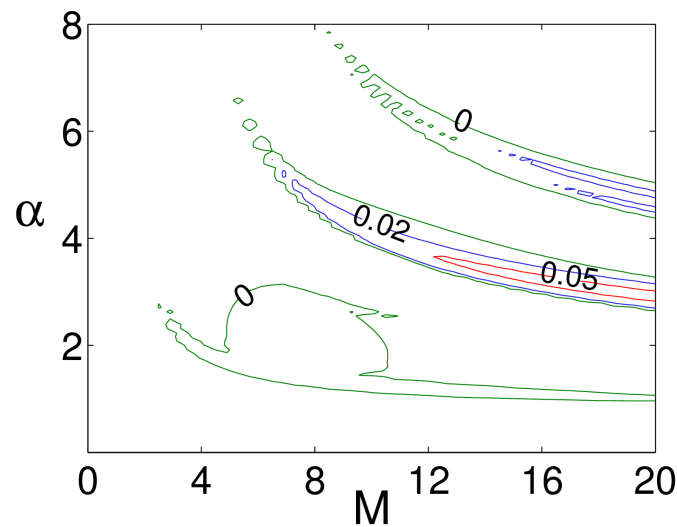
Stratification in conductivity,

- Similar maximum growth rate
- Less unstable regions

κ -averaged flow

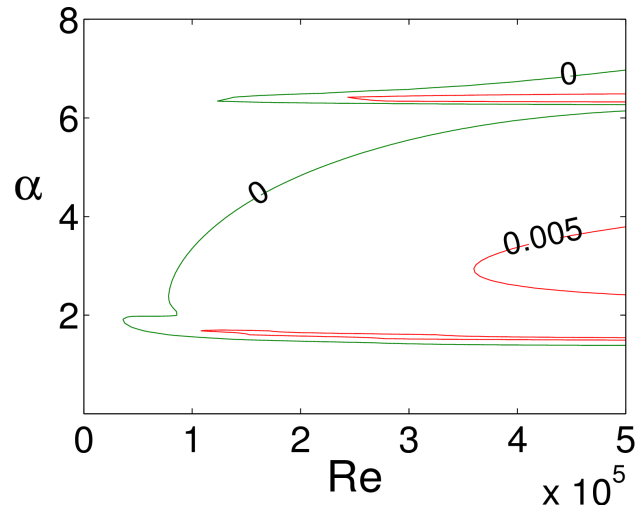


κ -stratified flow

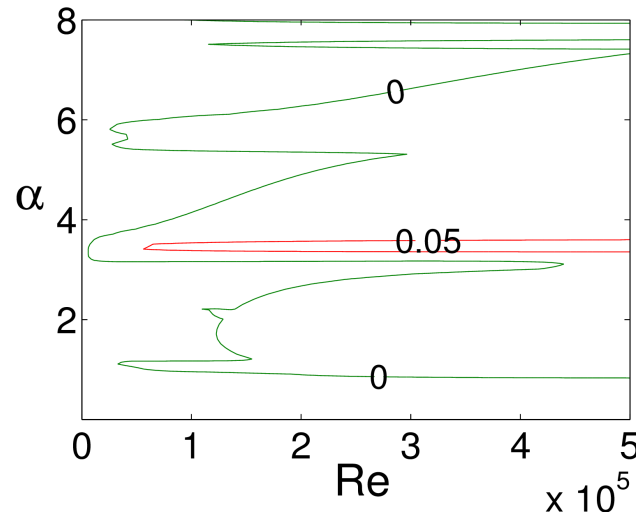


Effect of conductivity on critical Reynolds number

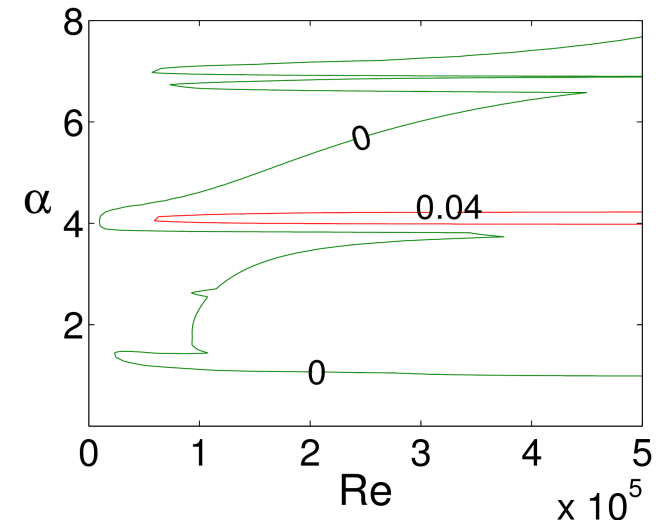
$M = 10, Pr = 0.72$



Uniform flow $\kappa = 1$



Averaged flow $\kappa = 3.42$

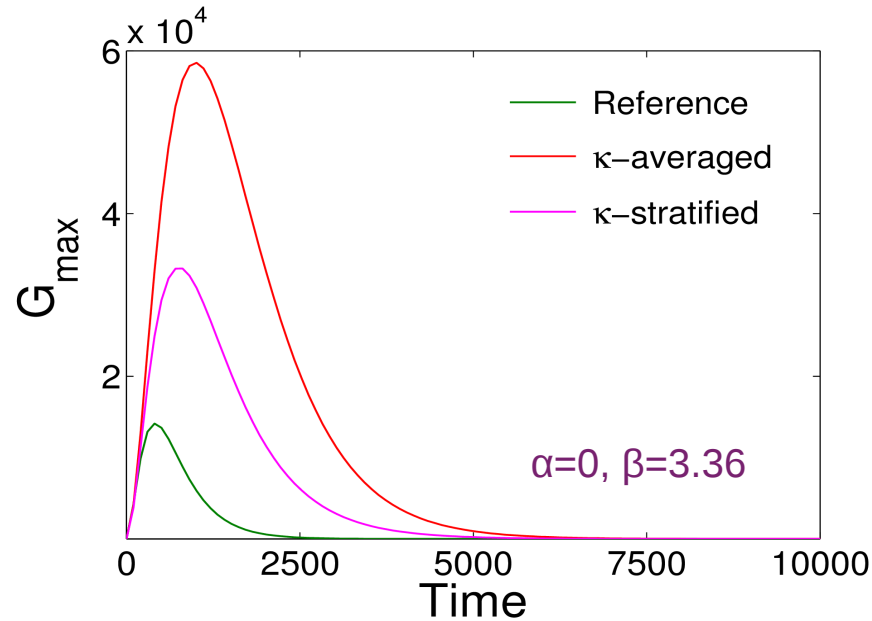


κ -stratified flow

Pr	M	Critical Reynolds number		
0.72		Reference	κ -averaged	κ -stratified
	3	40555	28051 ($\kappa_{avg} = 1.56$)	27426
	5	19527	13290 ($\kappa_{avg} = 2.13$)	12252
	10	36437	5537 ($\kappa_{avg} = 3.42$)	8860
	15	68620	7173 ($\kappa_{avg} = 4.55$)	10510

Effect of conductivity : Transient growth prediction

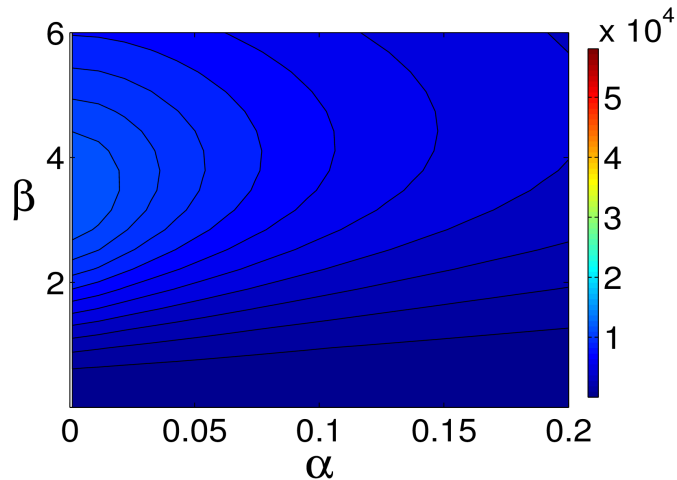
$M = 10$, $Re = 10^5$, $Pr = 0.7$



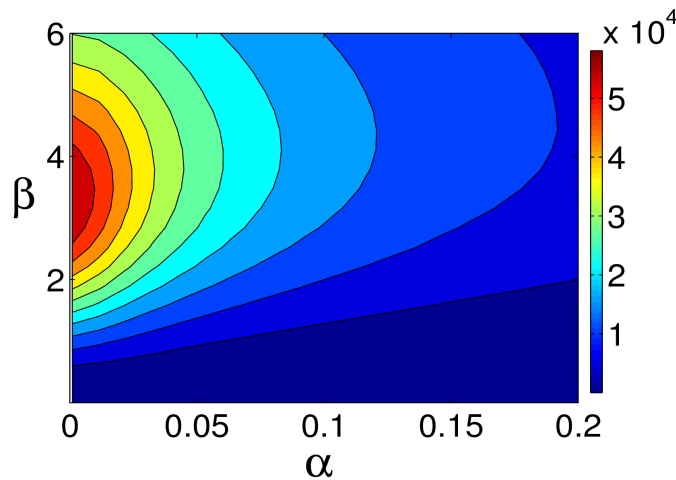
Strong destabilizing role of increase in the conductivity value

Stratification in conductivity is slightly stabilizing

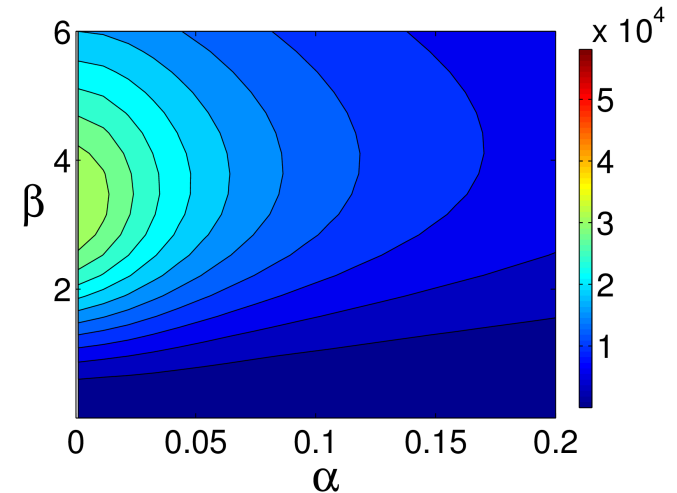
Reference flow



κ -averaged flow

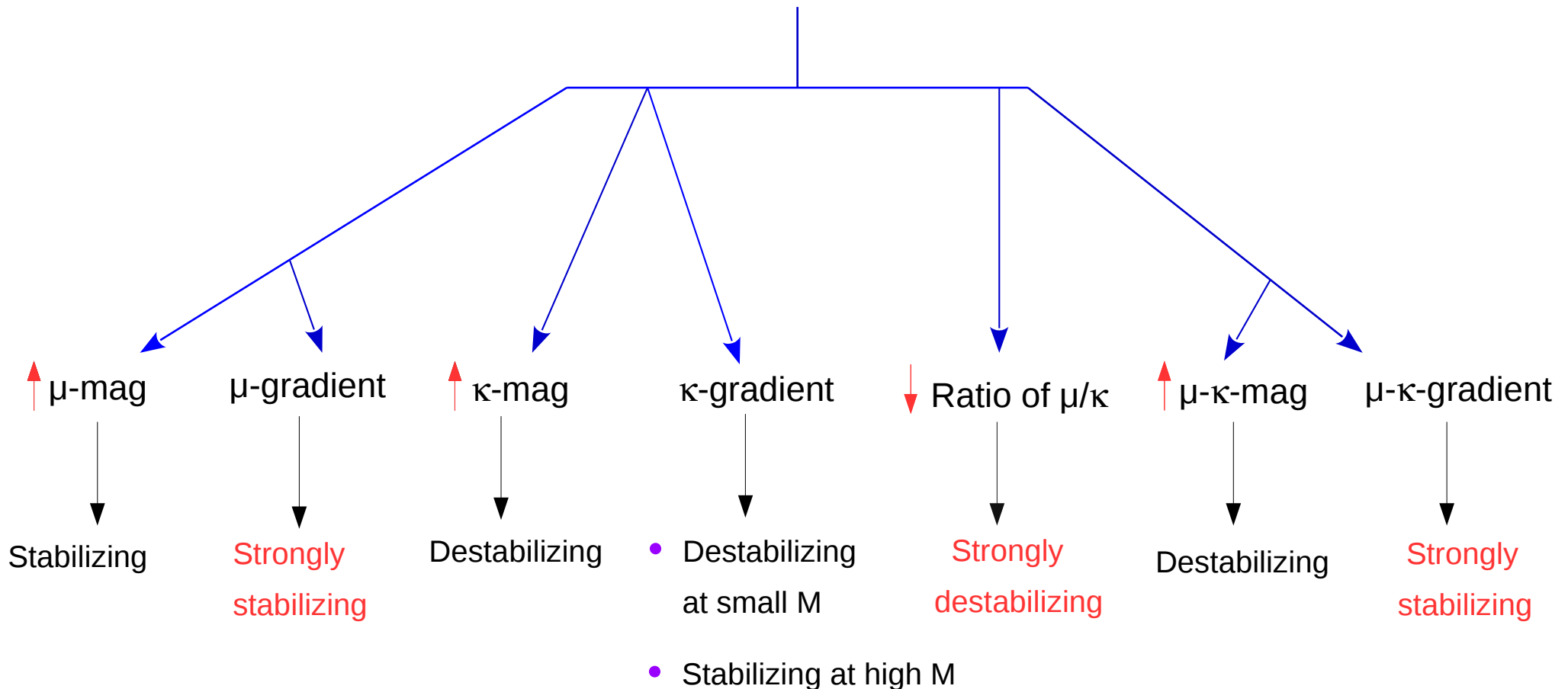


κ -stratified flow



Summary : Transport property effect

Transport properties



Non-modal energy budget

Mack's energy norm

$$2E = \int_0^1 \bar{\rho} (|u_s|^2 + |v_s|^2 + |w_s|^2) + \frac{\bar{T}}{\gamma \bar{\rho} M^2} |\rho_s|^2 + \frac{\bar{\rho}}{\gamma(\gamma-1)\bar{T}M^2} |T_s|^2 dy,$$

State vector

$$M = \text{diag}(\bar{\rho}, \bar{\rho}, \bar{\rho}, \frac{\bar{T}}{\gamma \bar{\rho} M^2}, \frac{\bar{\rho}}{\gamma(\gamma-1)\bar{T}M^2})$$

Total perturbation energy: Decomposition

$$\frac{\partial E}{\partial t} = -i \int_0^1 \phi^+ \mathbf{M} B \phi dy + c.c. = \frac{\partial P}{\partial t} + \frac{\partial V}{\partial t} + \frac{\partial T}{\partial t} + \frac{\partial S}{\partial t} + \frac{\partial W}{\partial t}$$

P VD TD SW PE

State vector

$$\phi = \sum_{k=1}^n \chi_k(0) \exp(-i\omega_k t) \psi_k$$

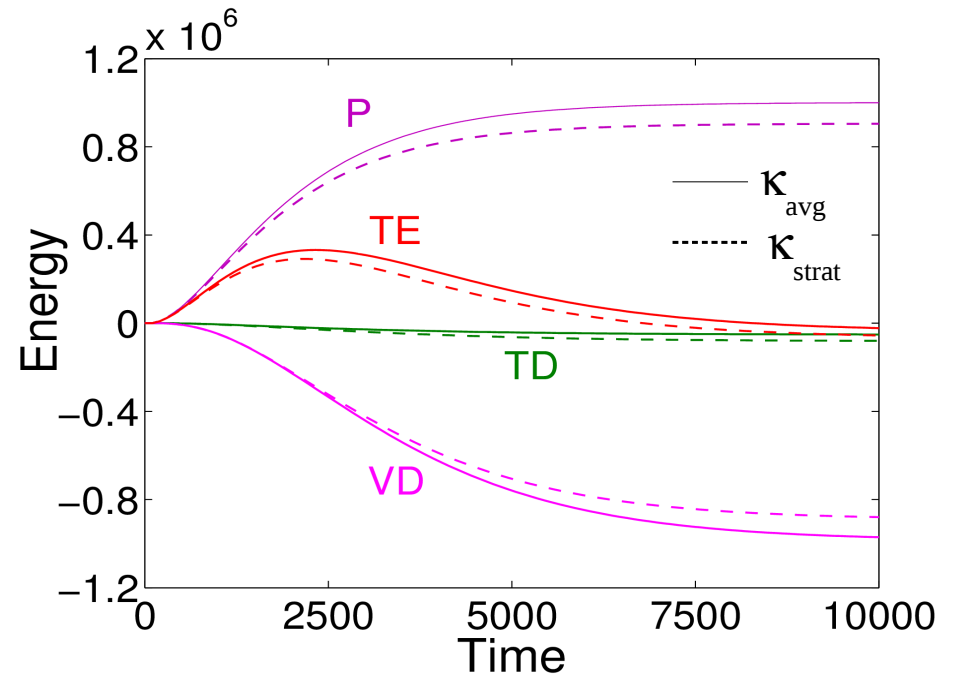
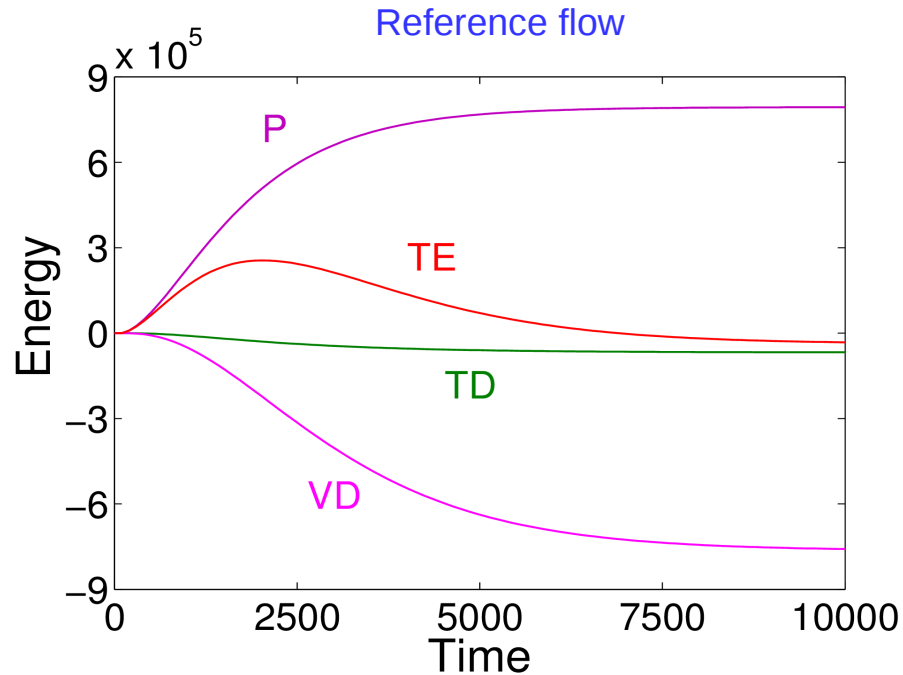
$$\frac{\partial P}{\partial t} = - \int_0^1 \left[\bar{\rho} \bar{U}_y u_s^+ v_s + \frac{\bar{T}}{\gamma M^2 \bar{\rho}} \bar{\rho}_y \rho_s^+ v_s + \frac{\bar{\rho}}{\gamma(\gamma-1) M^2 \bar{T}} \bar{T}_y T_s^+ v_s \right] dy + c.c.$$

$$\frac{\partial V}{\partial t} = - \frac{1}{Re} \int_0^1 [\alpha^2 (\bar{\mu} + \bar{\lambda}) u_s^+ u_s + \bar{\mu} (\alpha^2 + \beta^2) u_s^+ u_s - u_s^+ (\bar{\mu}_y D + \bar{\mu} D^2) u_s + \dots] dy + c.c$$

$$\frac{\partial T}{\partial t} = \frac{\bar{\rho}}{\gamma(\gamma-1) Re Pr M^2} \int_0^1 [\bar{\kappa}_T \bar{T}_{yy} T_s^+ T_s + \bar{T}_y^2 \bar{\kappa}_{TT} T_s^+ T_s + 2 \bar{T}_y \bar{\kappa}_T T_s^+ D T_s + \dots] dy + c.c$$

Transient energy budget: conductivity effect

$M = 5$, $Re = 10^5$, $Pr = 0.2$, $\alpha=0$, $\beta=3.4$



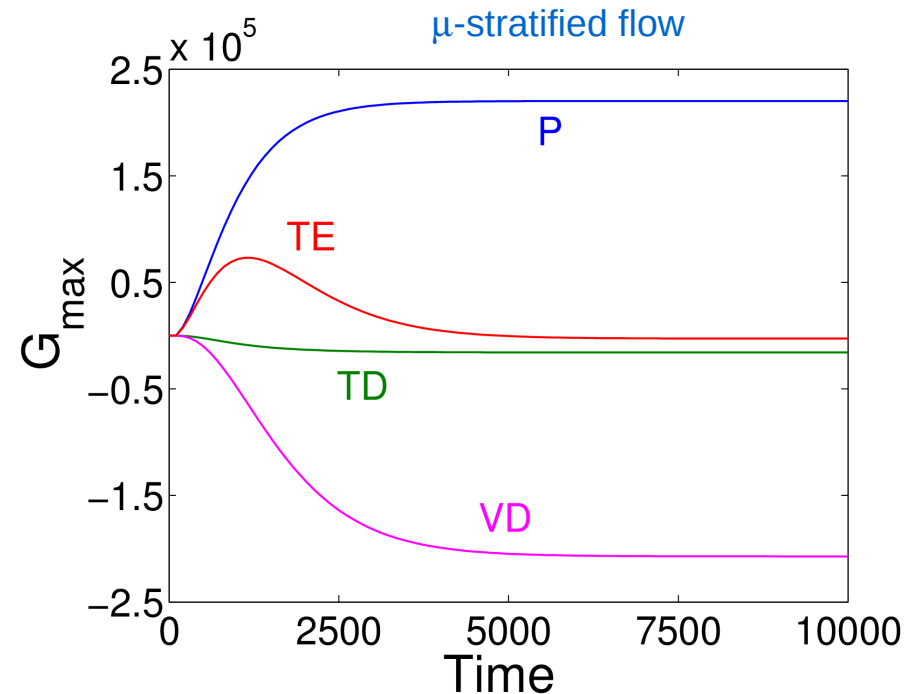
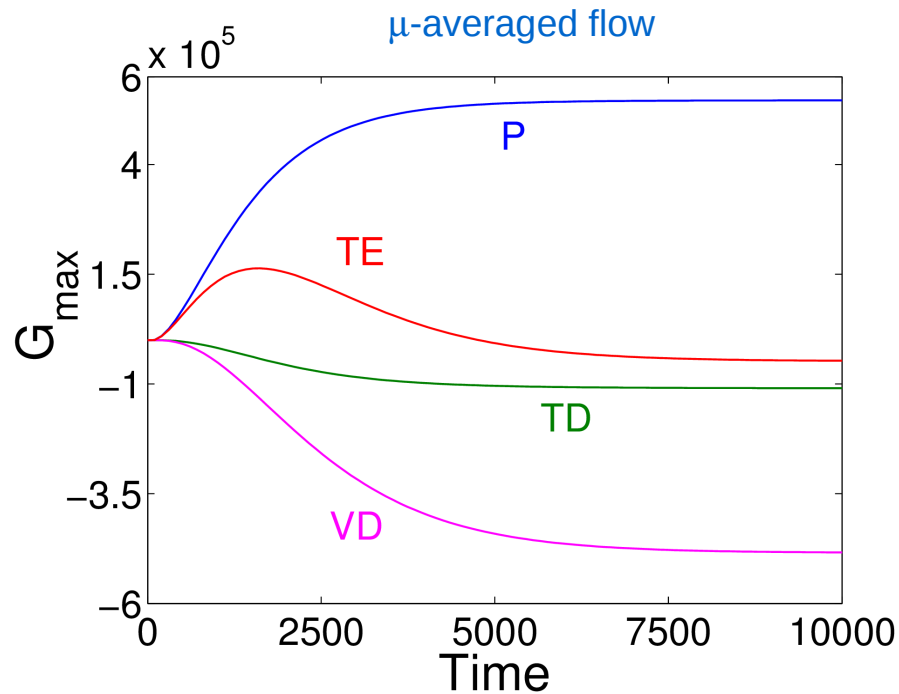
- Increase in conductivity,

- Main contributors viscous dissipation and production increase significantly
- Increases maximum value of total energy ~ 1.3 times

- Stratification in conductivity slightly reduces the energy transfer from the base flow

Transient energy budget: effect of viscosity stratification

$M = 5$, $Re = 10^5$, $Pr = 0.2$, $\alpha=0$, $\beta=3.3$

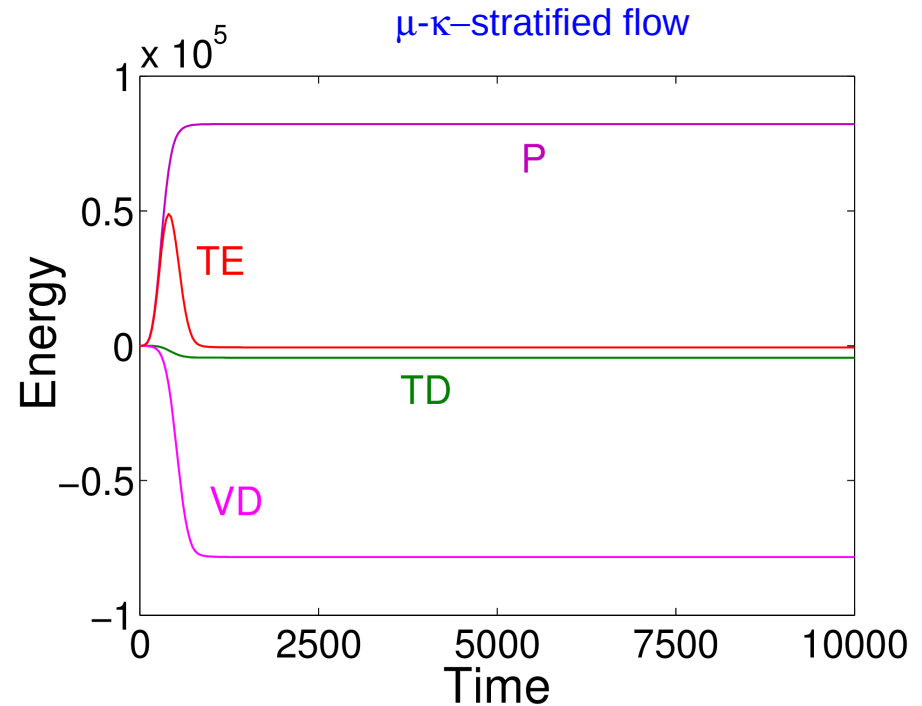
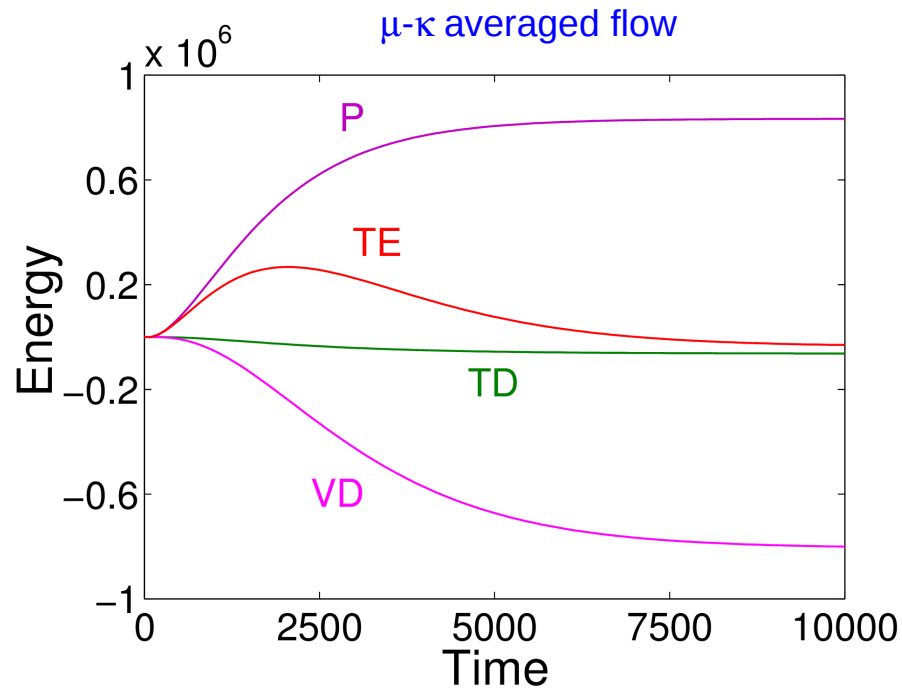


Viscosity stratification leads to,

- Significant reduction in production, viscous dissipation and thermal diffusion.
- Decrease in total energy by ~ 2.4 times.

Transient energy budget: combined effect of viscosity & conductivity stratification

$M = 5$, $Re = 10^5$, $Pr = 0.2$, $\beta = 3.5$



Stratification in viscosity & conductivity leads to,

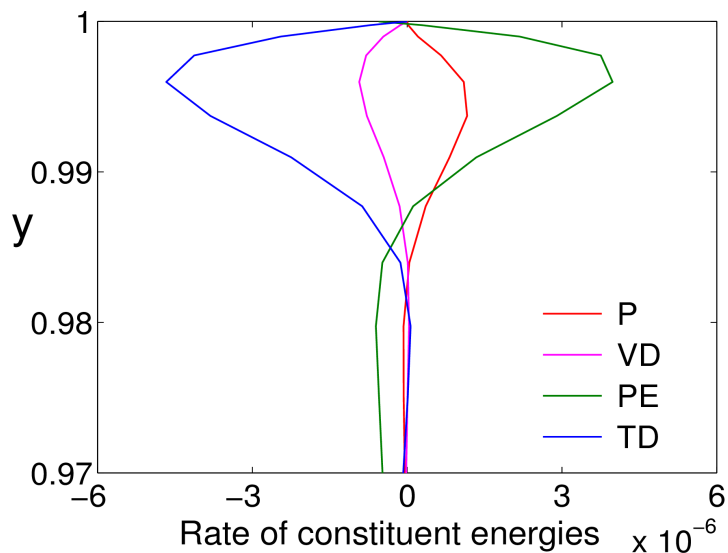
- Sharp variation at small time, reach the asymptotic values quickly
- Significant reduction in total energy by ~ 2.6 times

Reduction in transient energy amplification

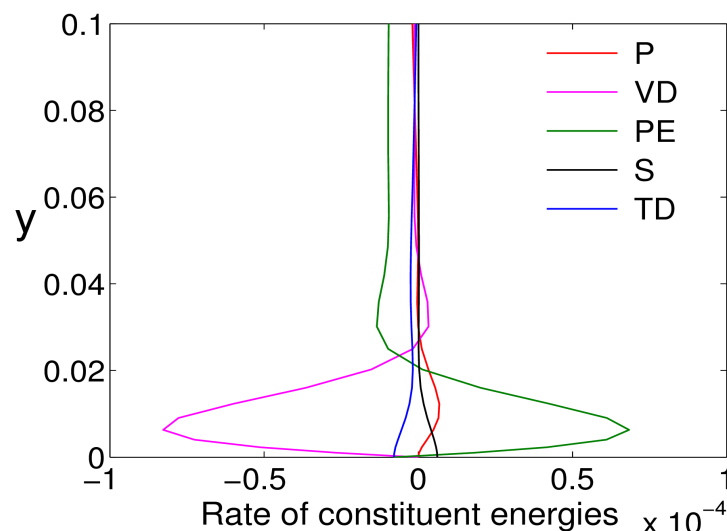
LST energy budget: specific to modes

$M = 10$, $Re = 10^5$, $Pr = 0.72$, $\alpha = 3$

Top wall



Bottom wall



Mode 1

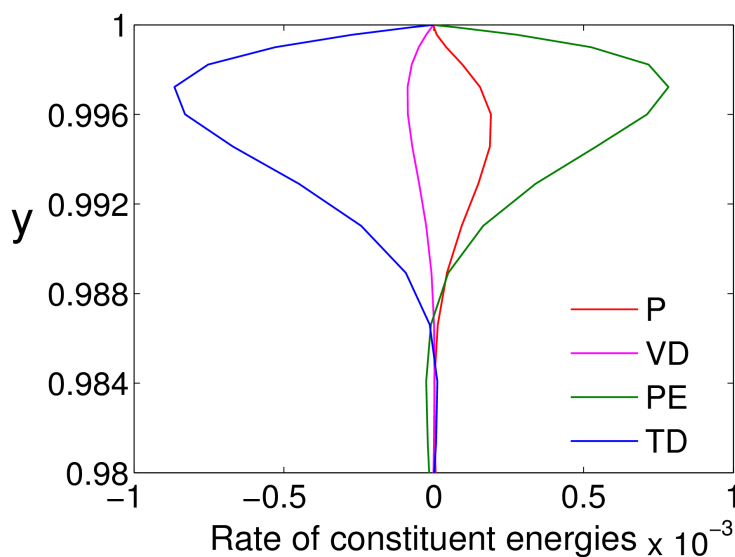
Energy transfer mechanisms are dominant near the **bottom** wall

The total energy transferred is dissipated and diffused

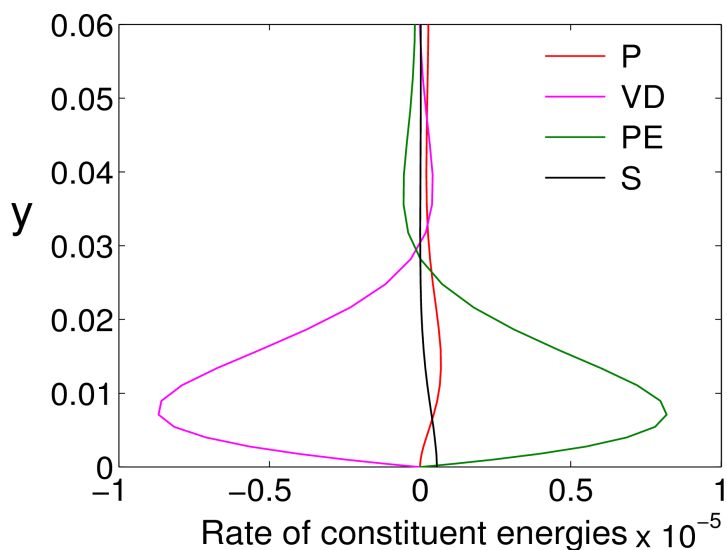


Flow is modally stable

Top wall



Bottom wall



Mode 2

Energy transfer mechanisms are dominant near the **top** wall

Mechanism leading to transient energy growth

- Lift up / Vortex tilting mechanism
- Orr / Reynolds stress mechanism

Depends on $r = \frac{\text{Spanwise wavenumber}}{\text{Streamwise wavenumber}} = \frac{\beta}{\alpha}$

$r = 0$ (i.e. $\beta = 0$) $0 < r < \infty$ $r = \infty$ (i.e. $\alpha = 0$)

Orr mechanism

Combined effect

Lift up

Energy amplification:

13 times

1185 times

1166 times (Butler & Farrell, POF, 4.8, pp 1637, 1992)

Time required:

8.7 advective time

117 time

138 time

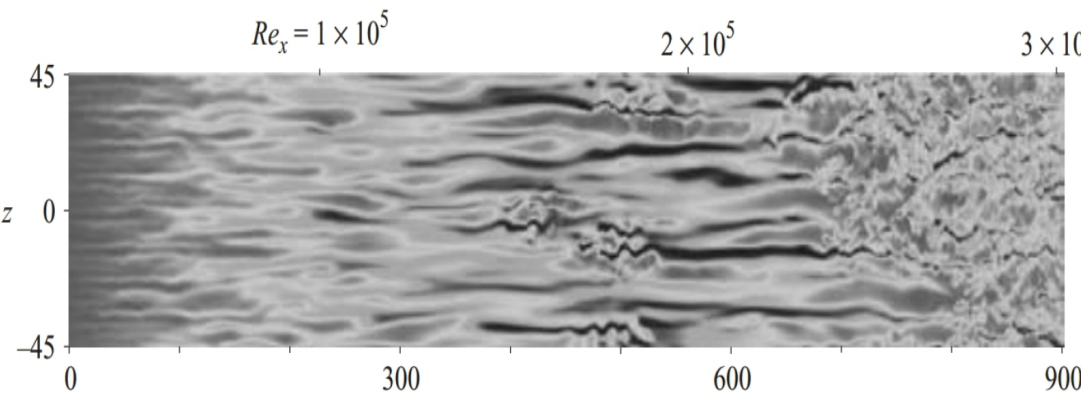
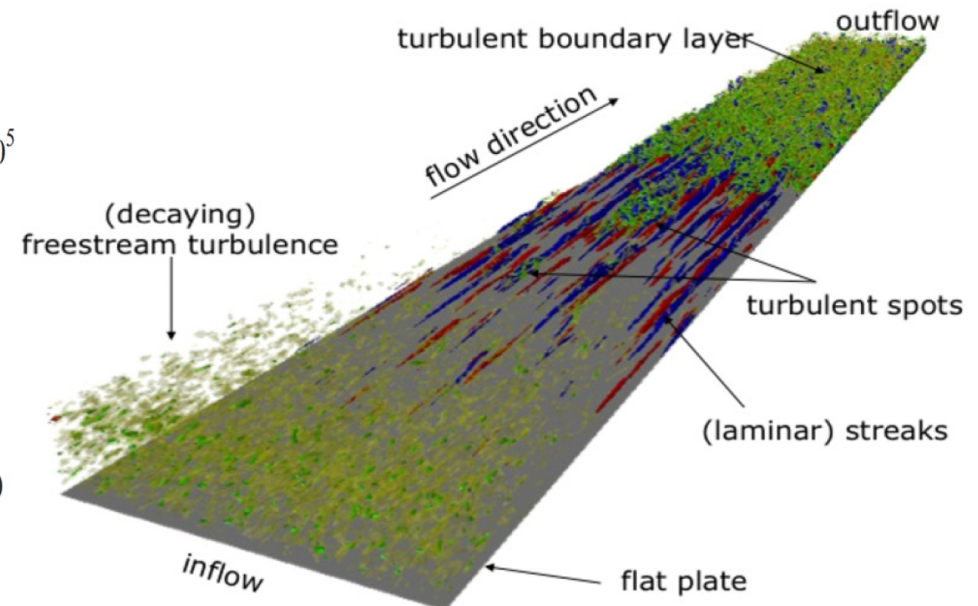


Fig: Instantaneous streamwise velocity

Brandt et. al. JFM, 517, pp. 167, 2004



Brandt Luca, EJM-B/Fluids 47, pp 80, 2014

Lift-up mechanism

Initial condition: Infinitely long streamwise vortices ($\alpha = 0$)

(most dangerous
for shear flows)

Streamwise vortices redistribute streamwise
momentum and energy

Streaks in velocity & temperature

Dominates in

- Reference
- κ -averaged
- μ -averaged
- κ -stratified
- μ -stratified

$$\hat{u} = \hat{u}_0 - \frac{d\bar{U}}{dy} \hat{v}_0 t$$

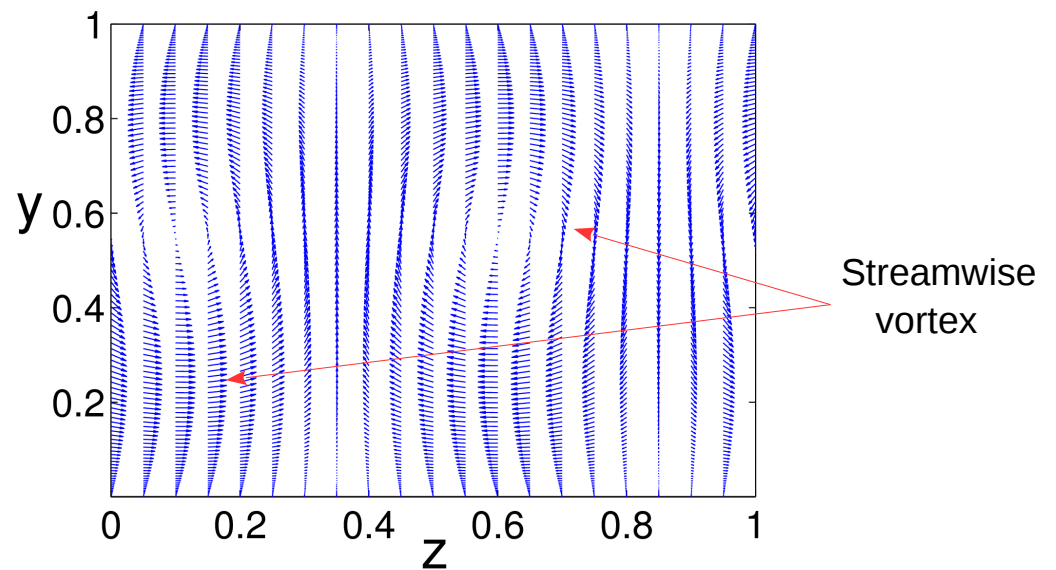
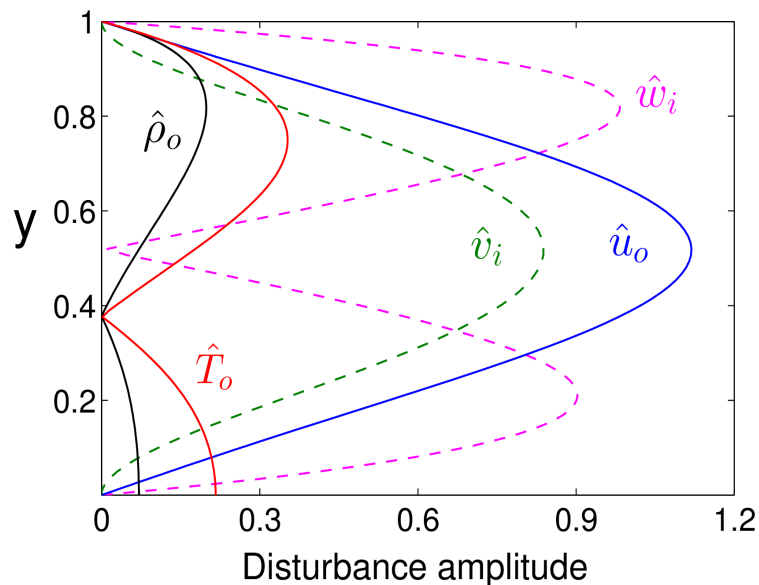
$$\hat{T} = \hat{T}_0 - \frac{d\bar{T}}{dy} \hat{v}_0 t$$

$\hat{u}$ remains bounded as $t \rightarrow \infty$

Streamwise extent of the
disturbed region grows

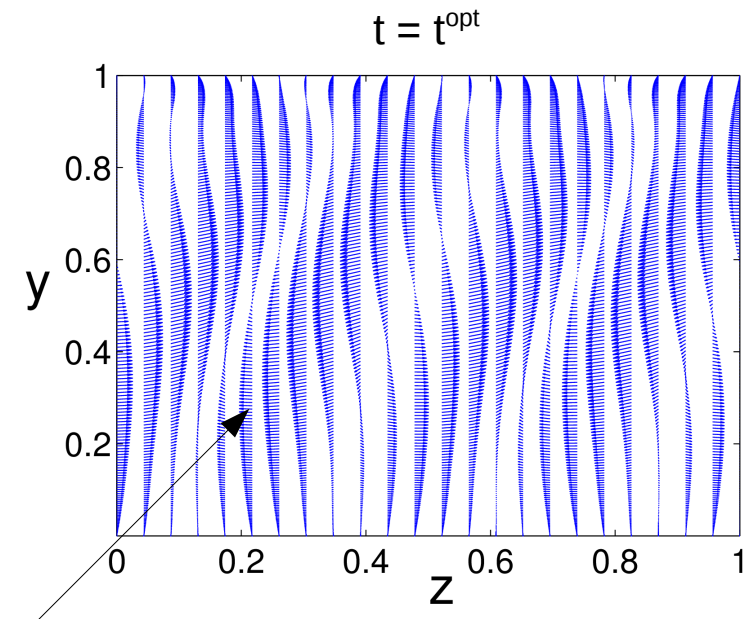
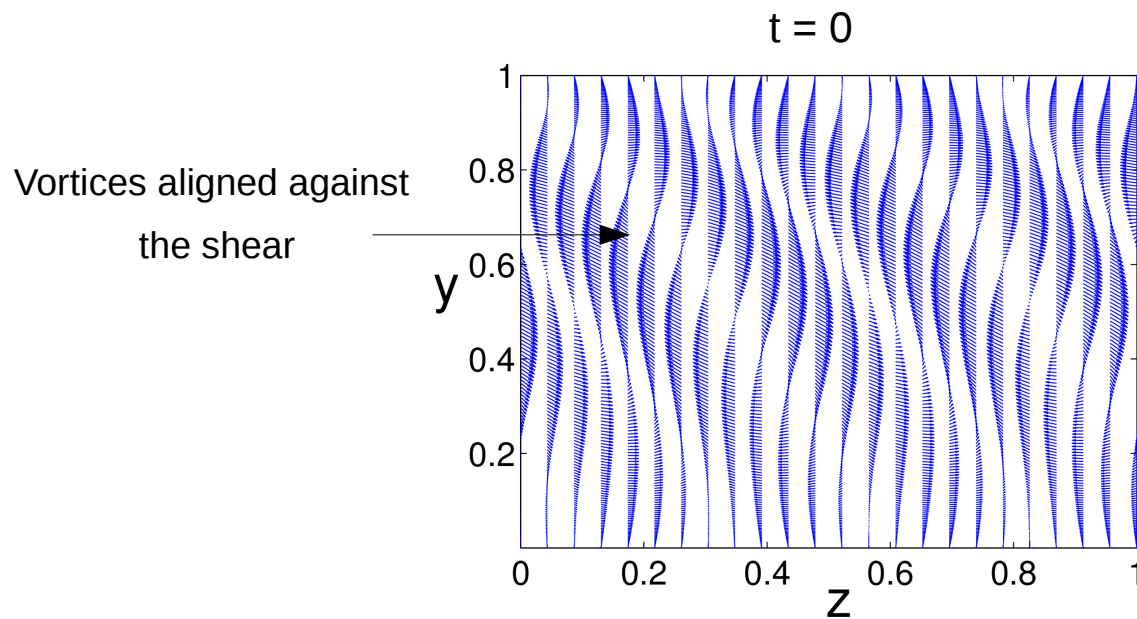
Energy grows linearly

κ -stratified flow



$$M = 5, \text{Re} = 10^5, \text{Pr} = 0.2, \beta = 3.6$$

Combined Orr + Lift-up mechanism : Viscosity and conductivity stratified flow



Orr mechanism:

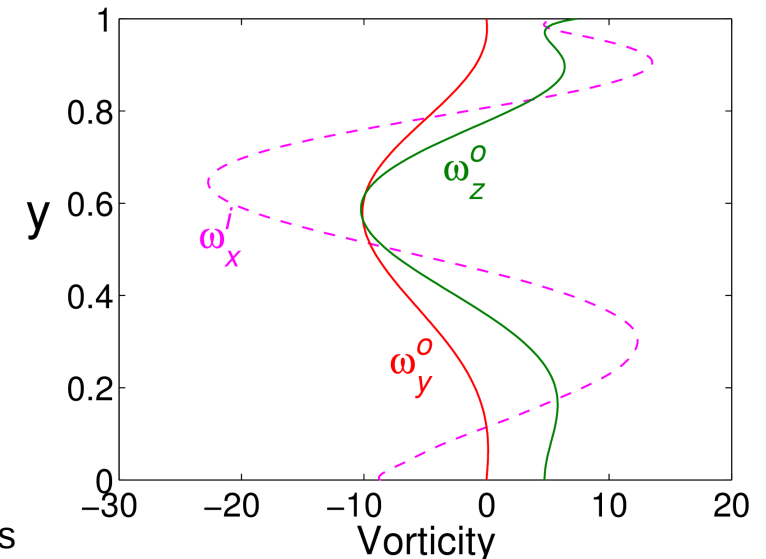
$$\frac{\partial E}{\partial t} = - \int \frac{d\bar{U}}{dy} \left(\left(\frac{\partial y}{\partial x} \right)_{\psi} \left(\frac{\partial \psi}{\partial \gamma} \right)^2 \right) dy$$

Vortices aligned with the shear

Energy grows if, $\frac{d\bar{U}}{dy} \left(\frac{\partial y}{\partial x} \right)_{\psi} < 0$

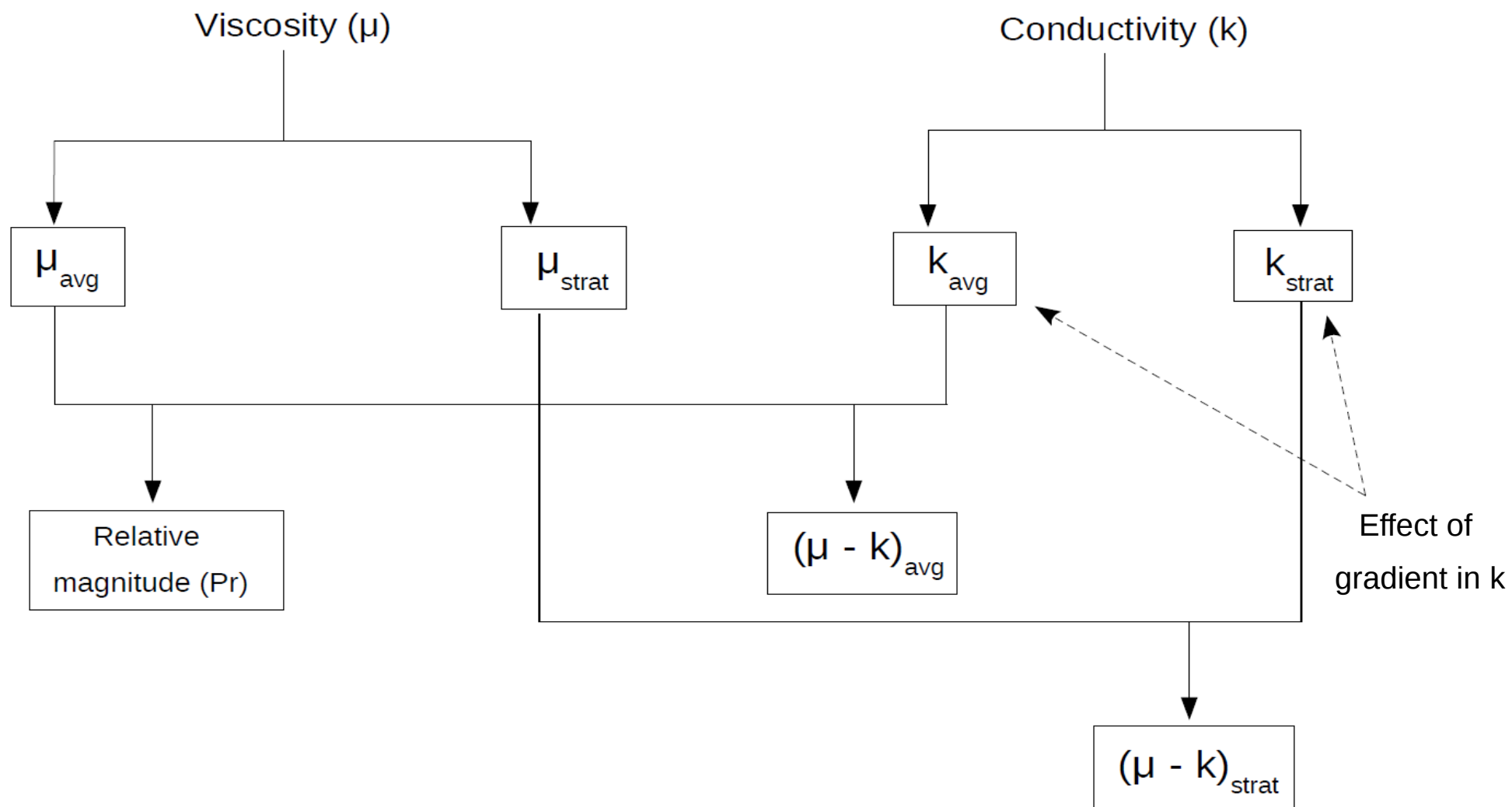
Extract the energy from mean flow → Align normal to the mean shear

Align along the mean shear ← Lose energy via Reynolds stress



$M = 10, Pr = 0.2, \alpha = 0.05, \beta = 3.5$

Thank you for listening !!



Thank you for listening !!