

Mechanics of a highly-bendable filament in the presence of a droplet

S.Ganga Prasath

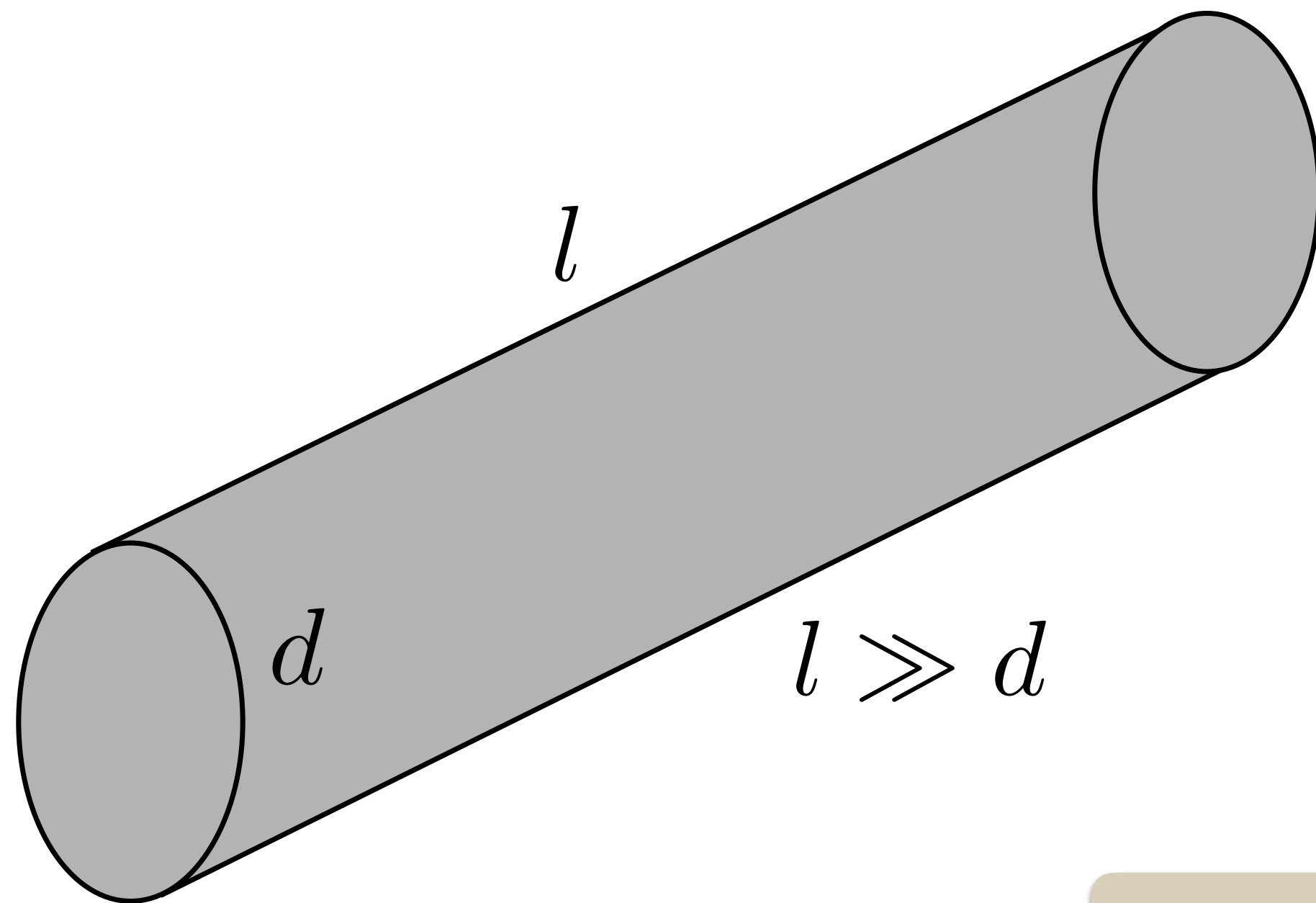
ICTS Bengaluru

guided by

Rama Govindarajan, Narayanan Menon, Benjamin Davidovich

What is a filament?

Any solid structure with very high aspect ratio



Mechanical properties

Young's modulus, E

Area moment of inertia, I

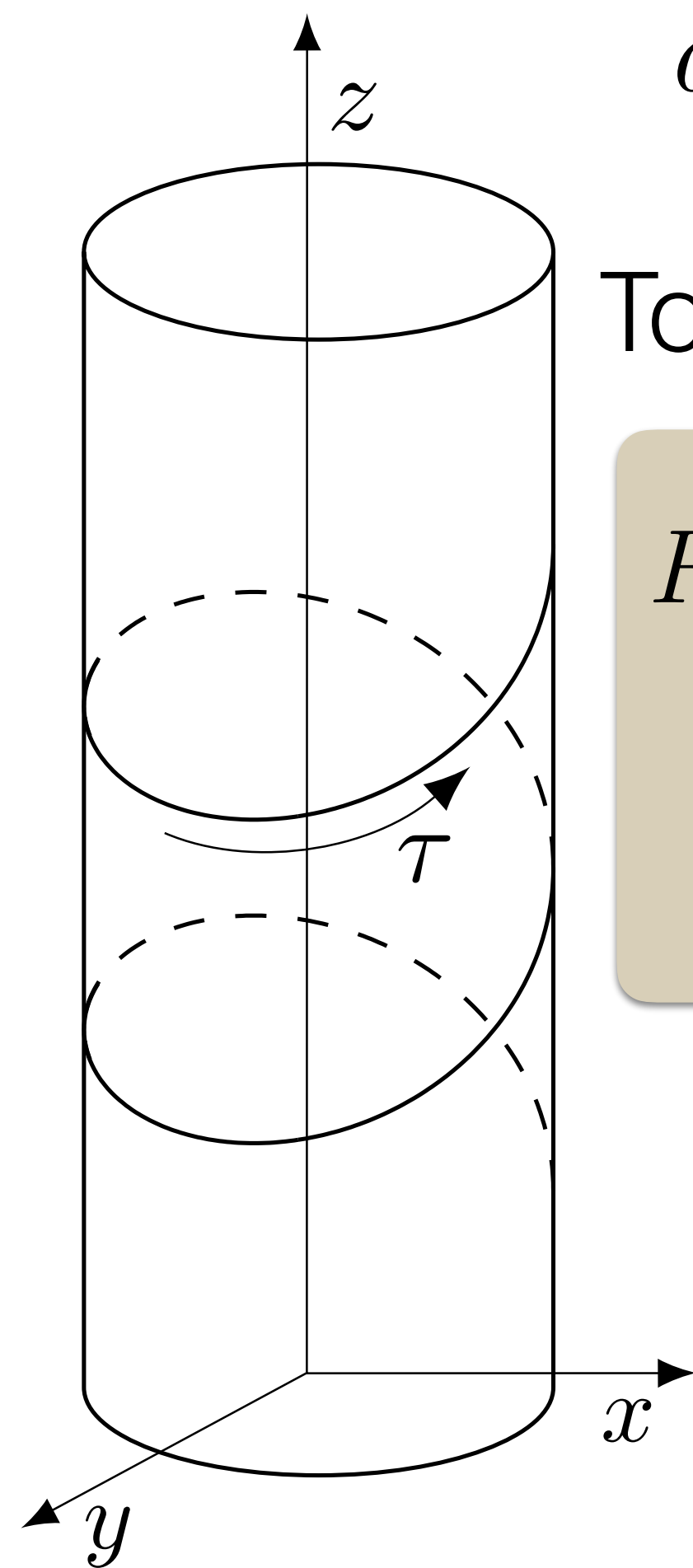
Inextensibility (Hookean limit)

$$\text{Axial strain, } \epsilon \sim \frac{\kappa}{1/l} \left(\frac{d}{l} \right)^2$$

Small as long as
 $d \ll l$

Different modes

Torsion



$$d\phi = \tau dz$$

Torsional energy density

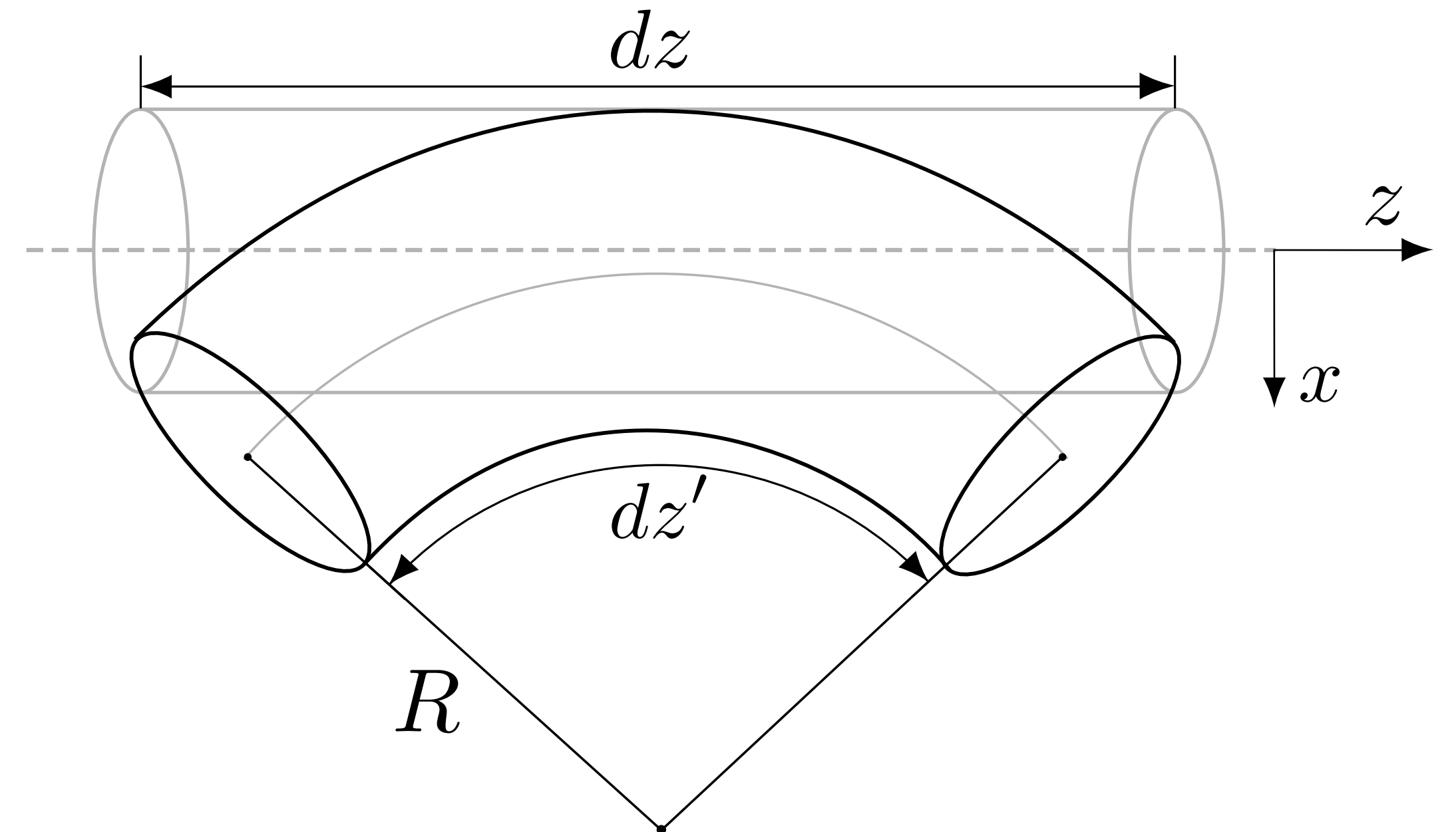
$$F_{tor} = \frac{1}{2} \int C \tau^2 dz$$

$$C = \int_S 4\mu (\nabla \chi)^2 dS$$

C – Torsional stiffness

$$C = \frac{\mu}{2} \pi \left(\frac{d}{2} \right)^4$$

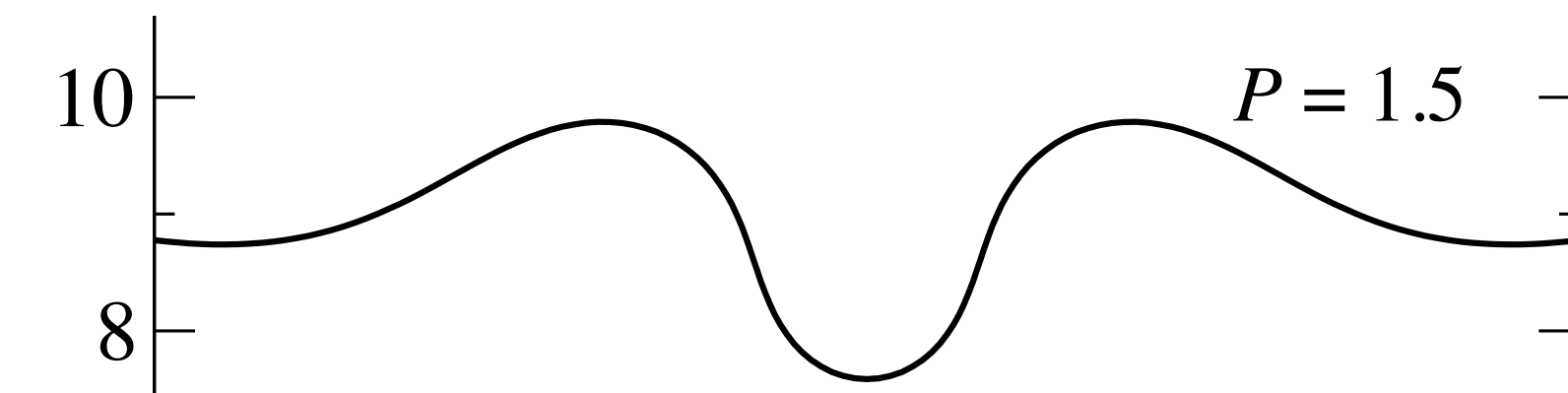
Bending



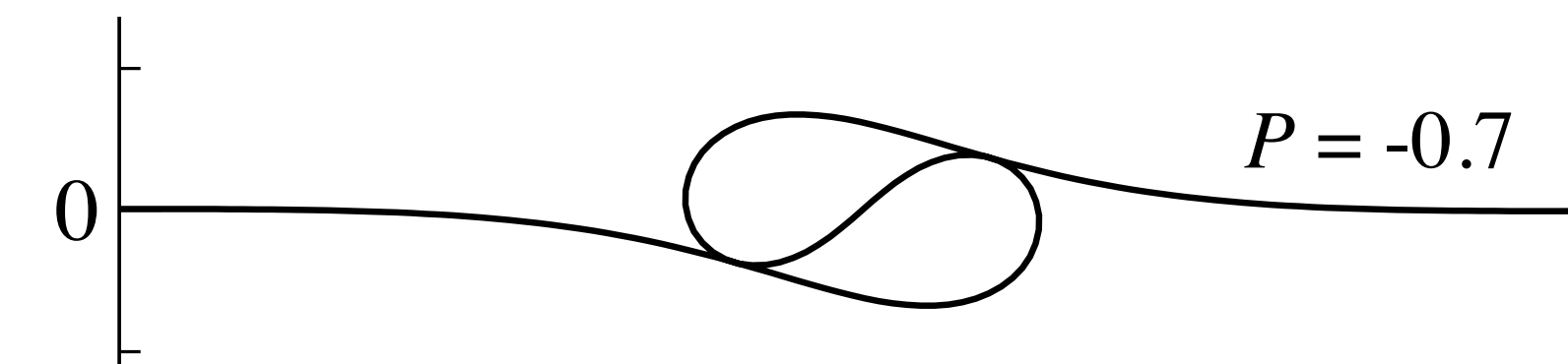
$$F_{ben} = \frac{1}{2} \sigma_{zz} u_{zz} = \frac{1}{2} \frac{E}{R^2} \int x^2 dS = \frac{1}{2} \frac{EI}{R^2}$$

$$M_y = \int x \sigma_{zz} dS = \frac{E}{R} \int x^2 dS = \frac{EI_y}{R}$$

Instabilities on filament



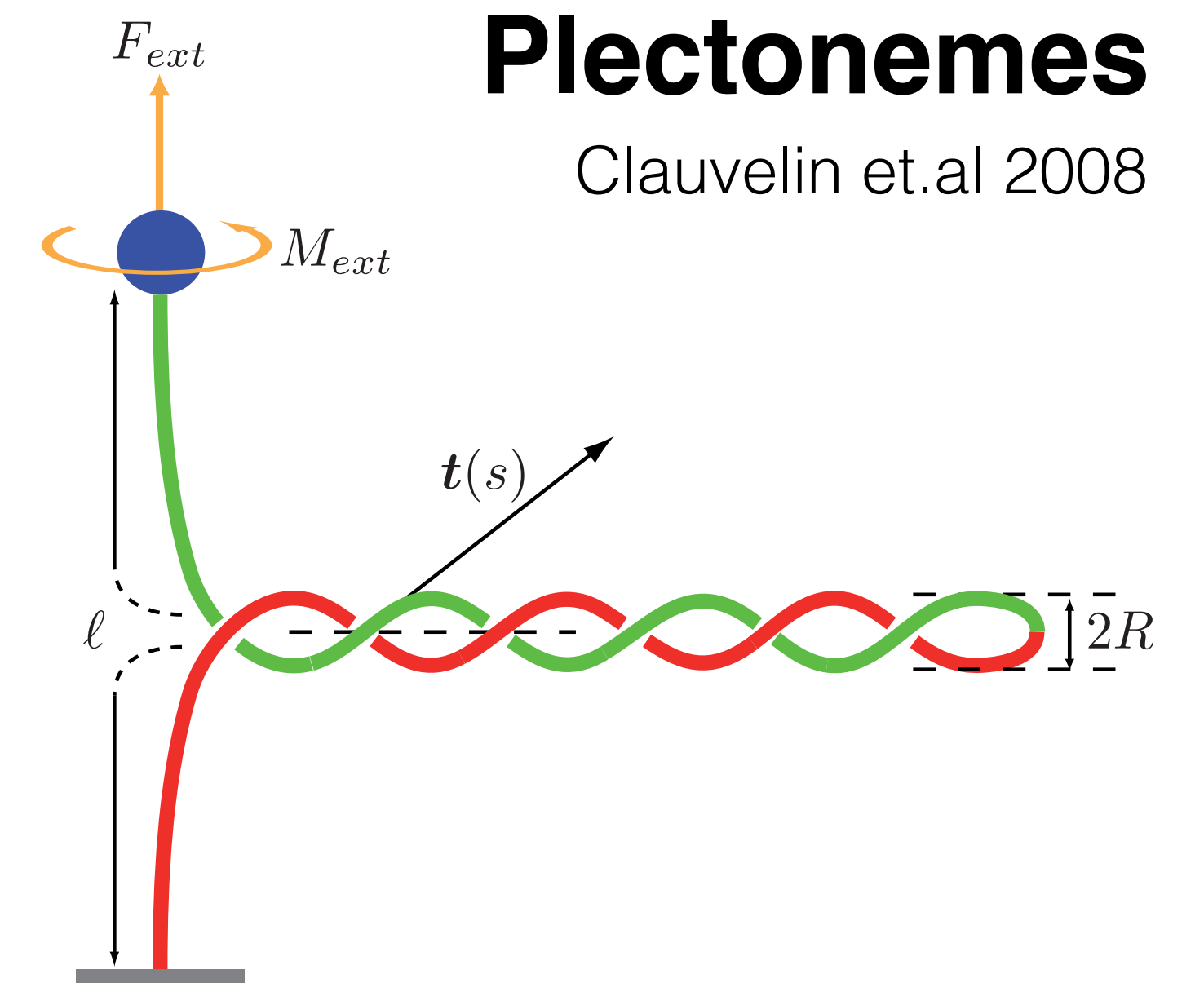
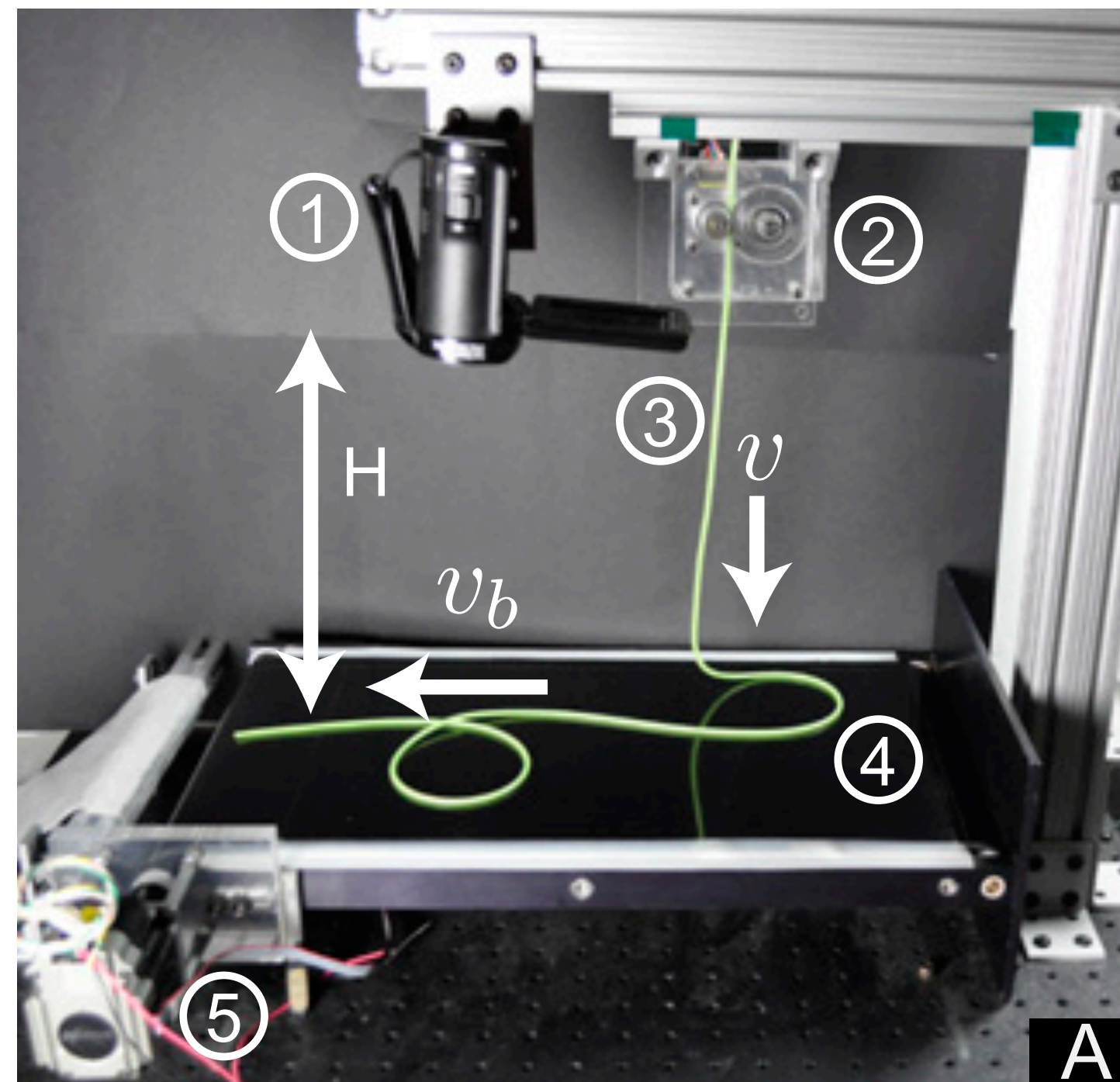
Buckling



Folding

Diamant et.al 2011

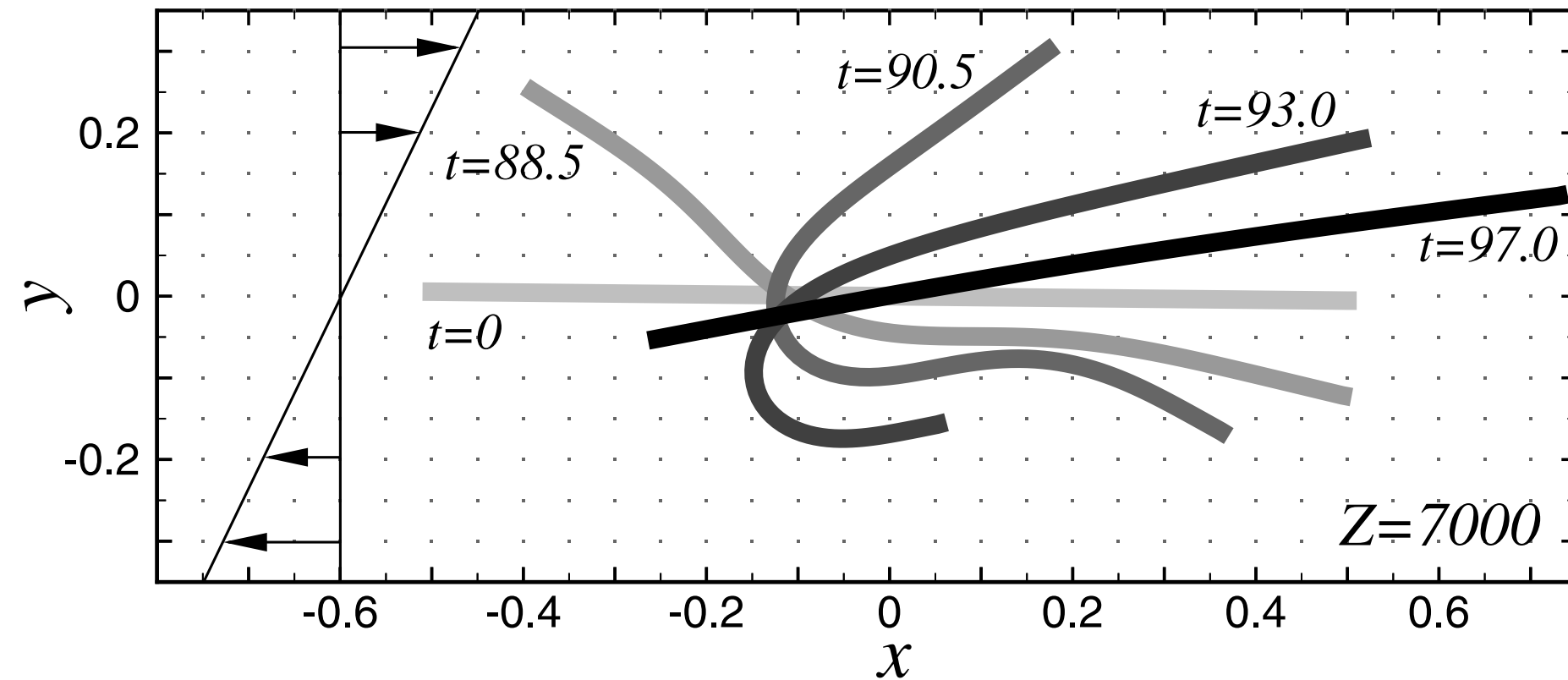
Coiling



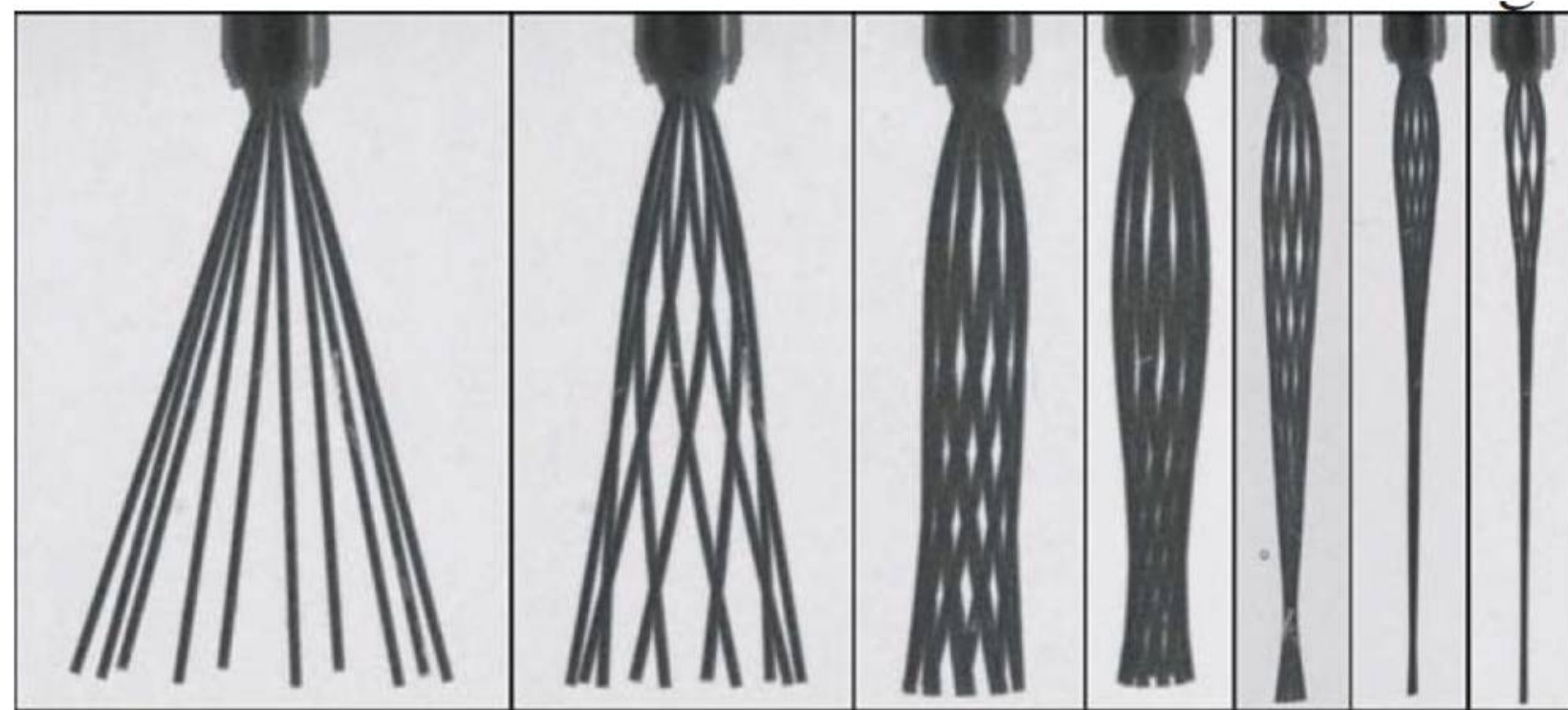
Jawed et.al 2014

Presence of fluid

Becker et.al 2001

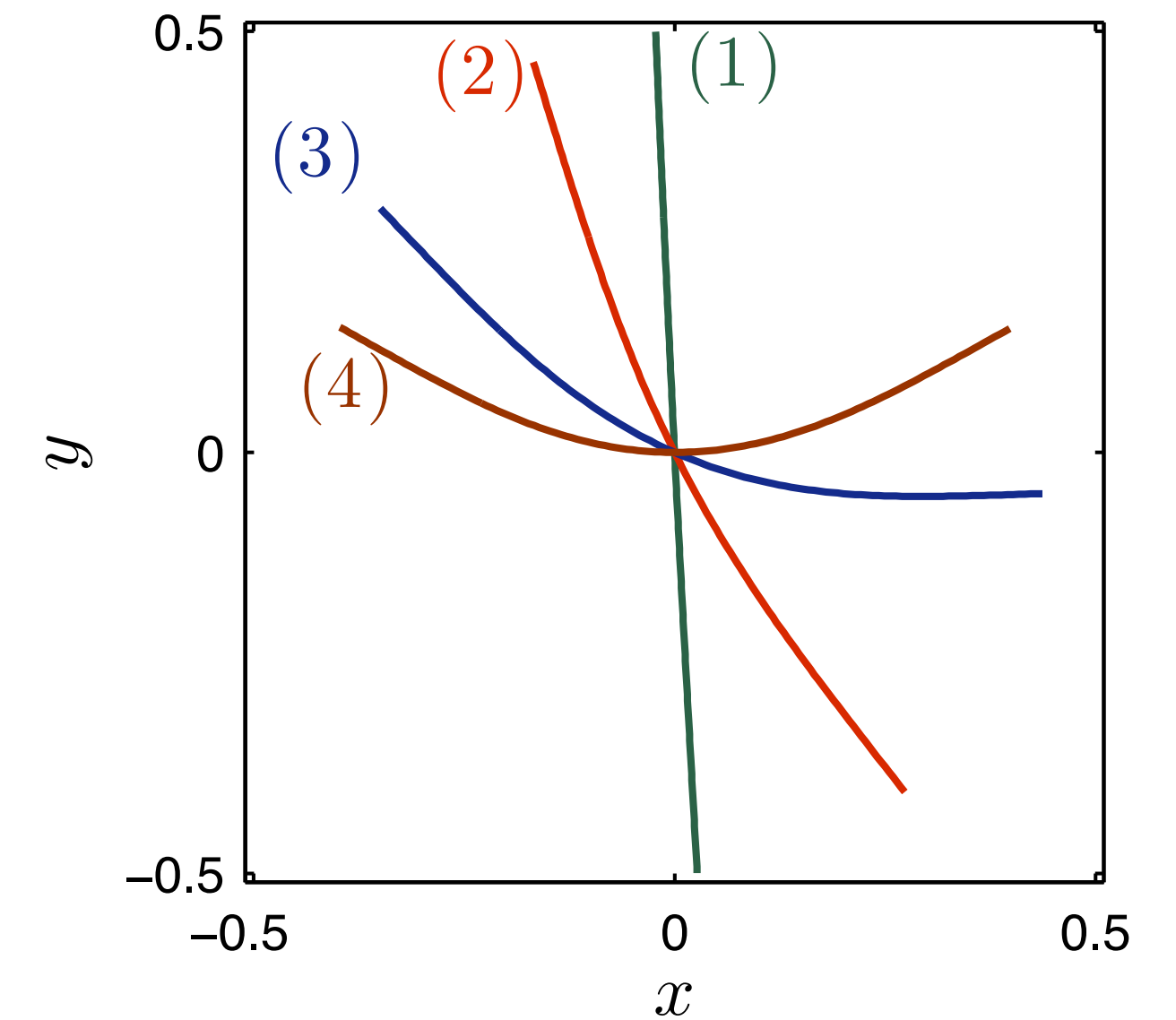


Buckling in shear flow



Folding due to rotation

Coq et.al 2008



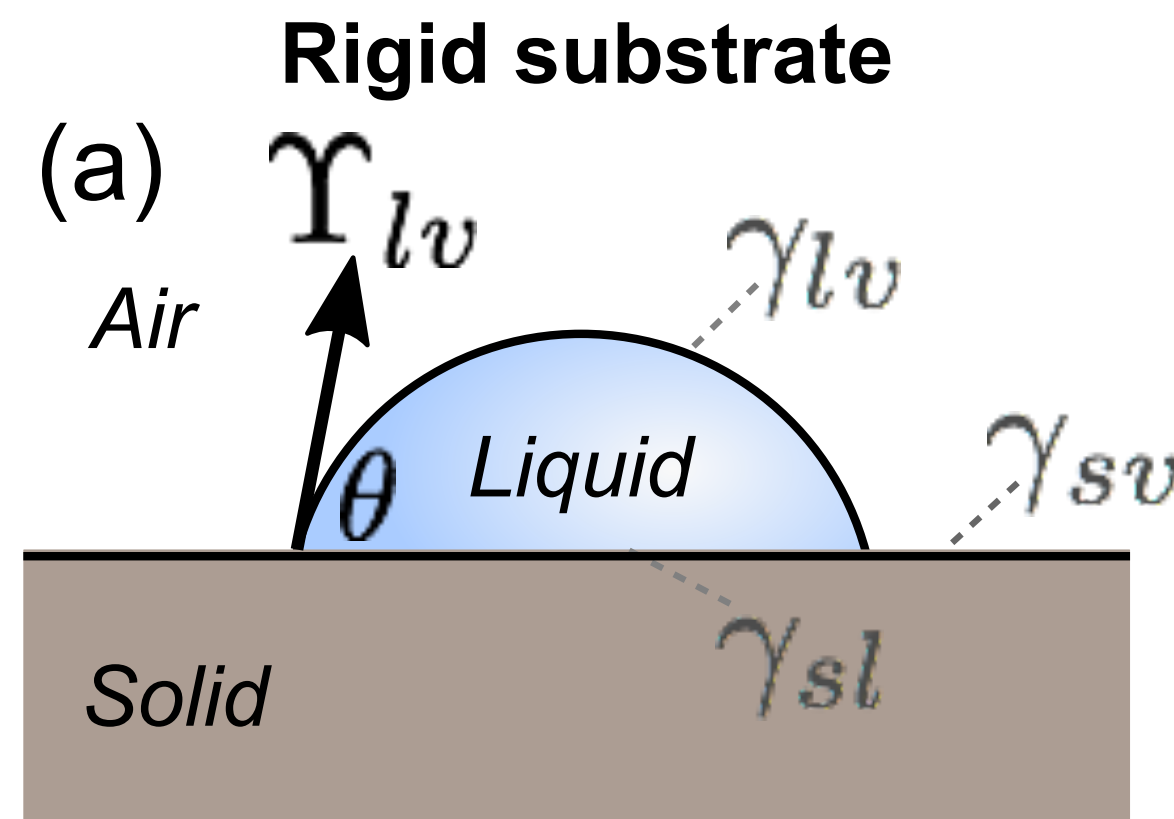
Sedimenting filament

Li et.al 2013

Coalescence of wet hair

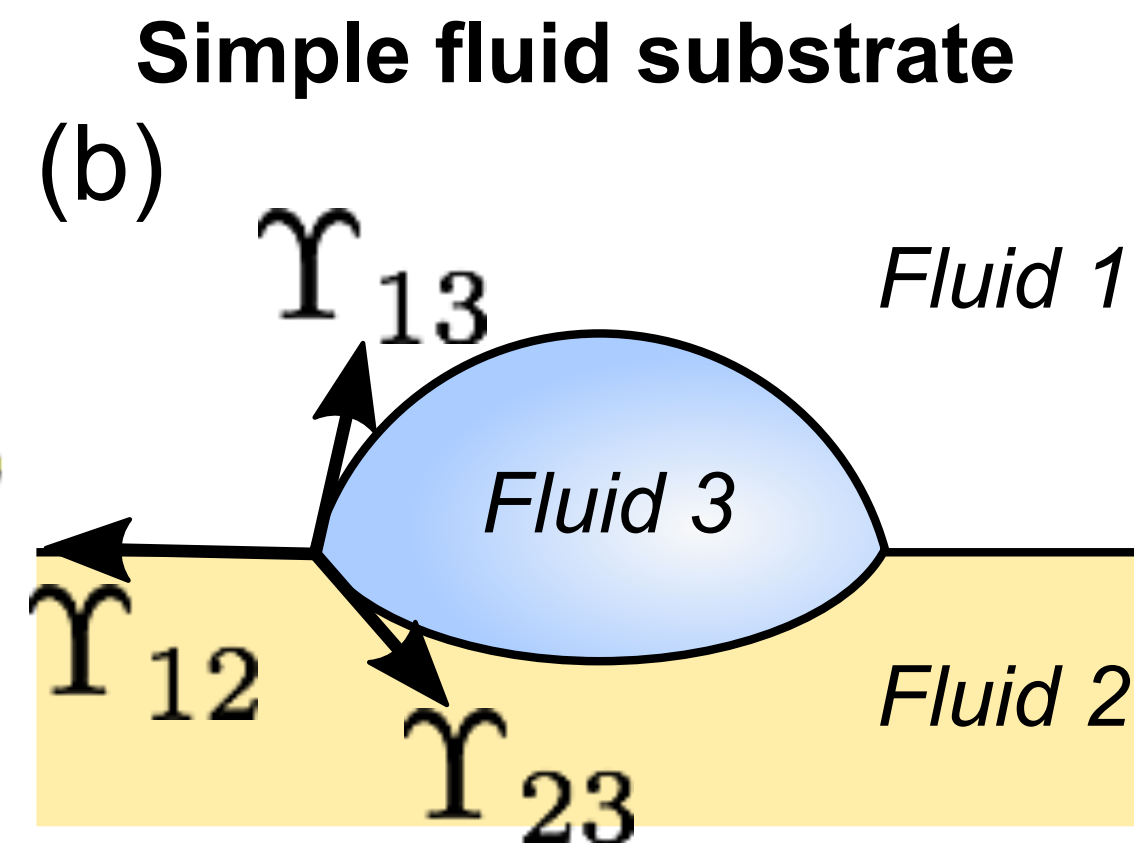
Bico et.al 2004

Contact angle



Young's type

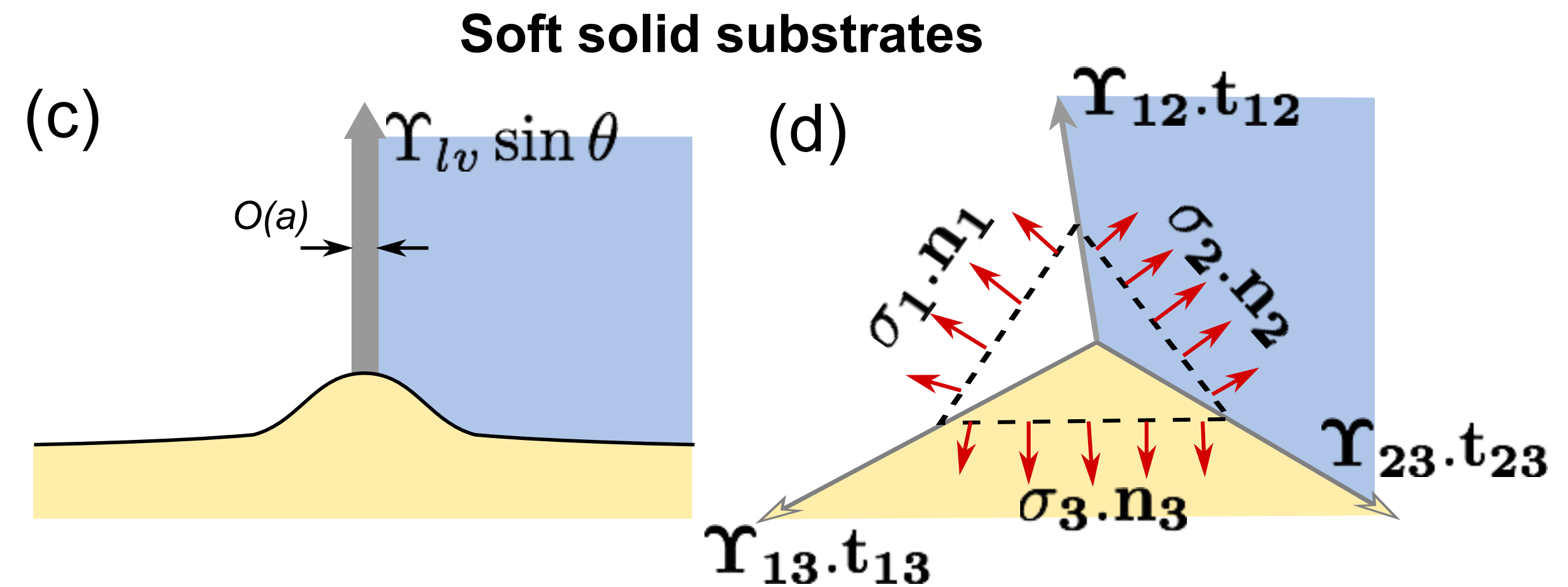
$$\cos(\theta_Y) = \frac{\Delta\gamma_s}{\gamma}$$



Neumann's type

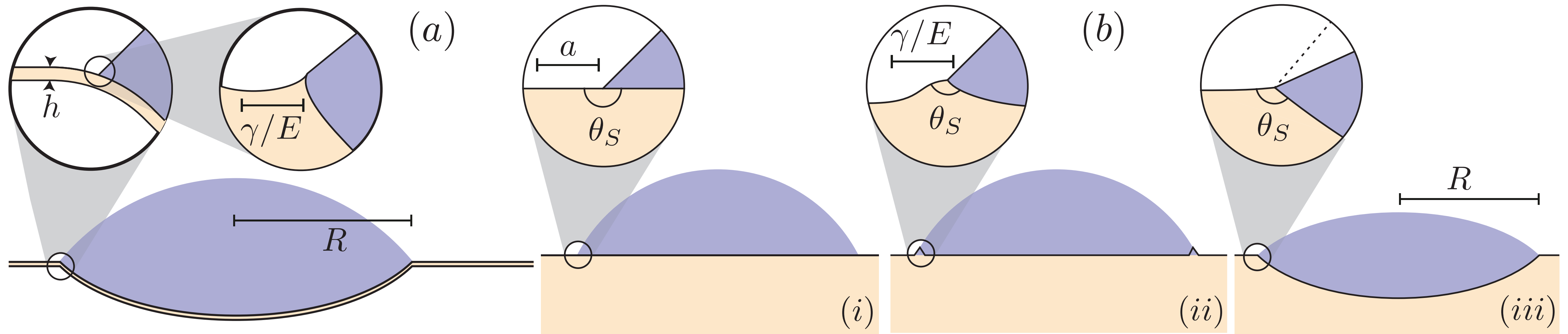
$$\cos(\theta_N) = \frac{1}{2} \left[\frac{\Delta\gamma_s}{\gamma} + \frac{\gamma}{\gamma_{sv}} \right],$$

$$\sin(\phi_N) = \frac{\gamma}{\gamma_{sl}} \sin(\theta_N).$$



Found to be Neumann's type

Length scales



Contact angle Young's or Neumann's type??

$$\gamma/E \ll a$$

$$a \ll \gamma/E \ll R$$

$$R \ll \gamma/E$$

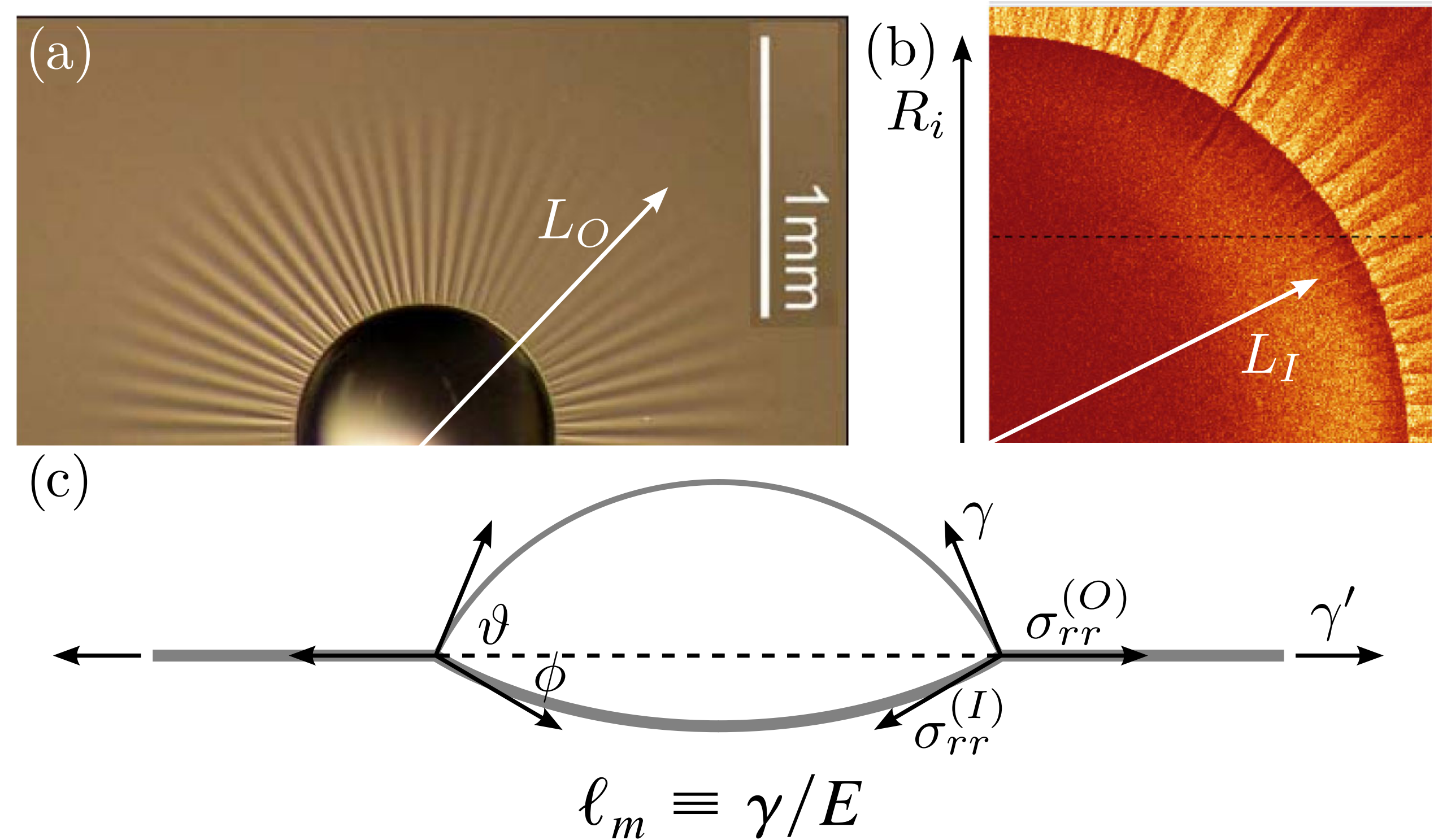
range of molecular interactions a

Thin sheets

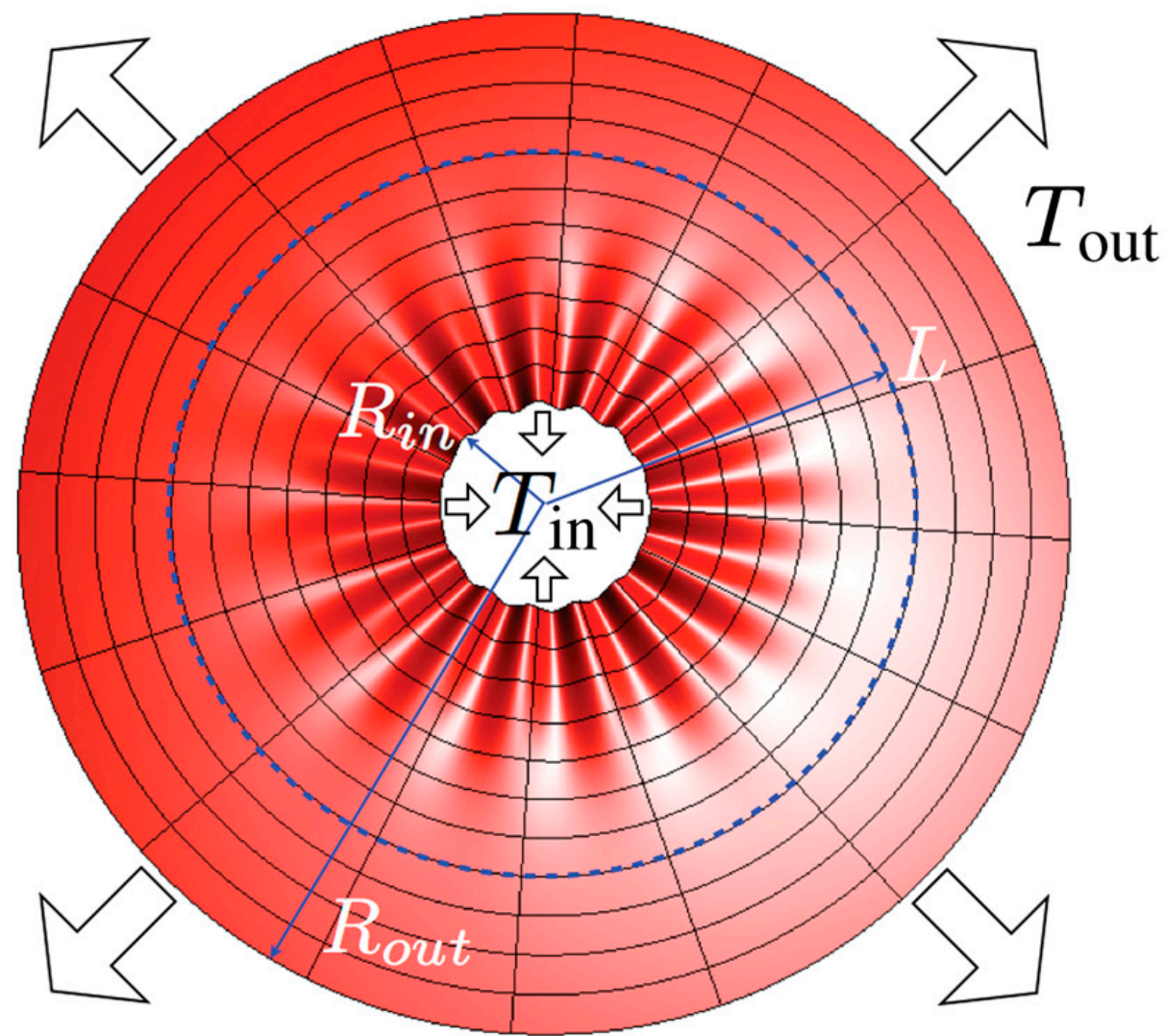
- Jump in stress at the contact line
- Contact angle of droplet on top of a sheet different from that on solid

$$\Delta\vartheta \sim (\ell_m/t)^{1/3}, \quad \ell_m = \gamma/E$$

- Gives length of the wrinkles



Model

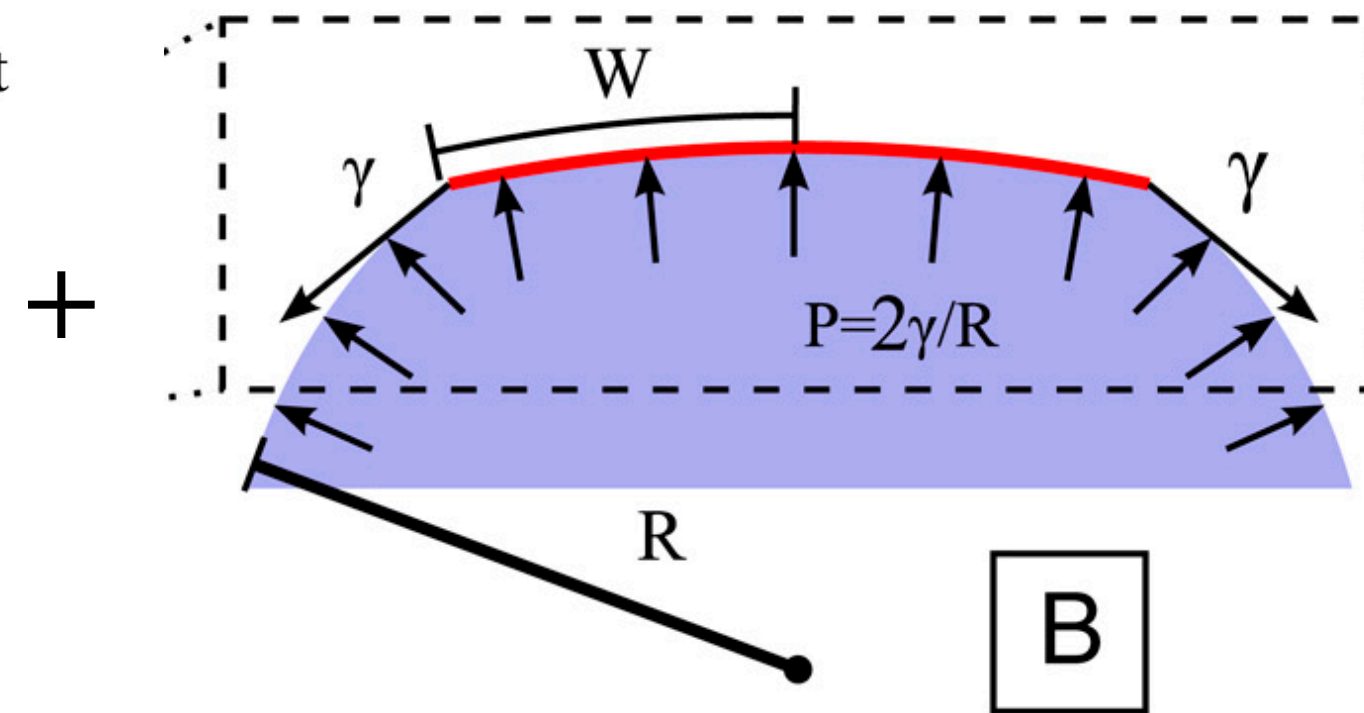


Lame's problem

Bendability

$$\epsilon^{-1} = \gamma R^2 / B ,$$

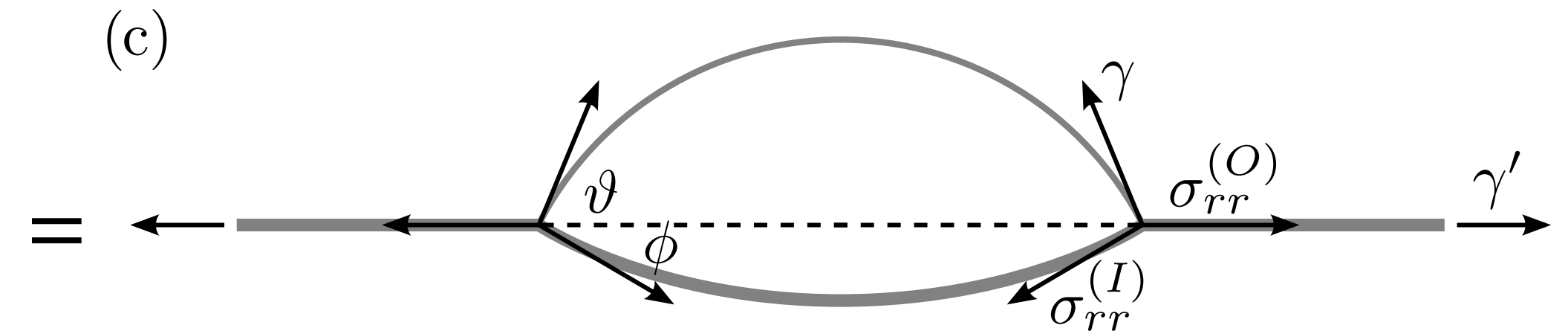
How strong is surface tension w.r.t bending?



Sheet over drop

$$U_{\text{total}} = U_{\text{elastic}} + \gamma_{sl'} A_{\text{bath/sheet}} + \gamma_{sl} A_{\text{sheet/drop}} + \gamma_{sv} A_{\text{sheet/vapor}}$$

Minimise energy with volume conservation



Drop on sheet

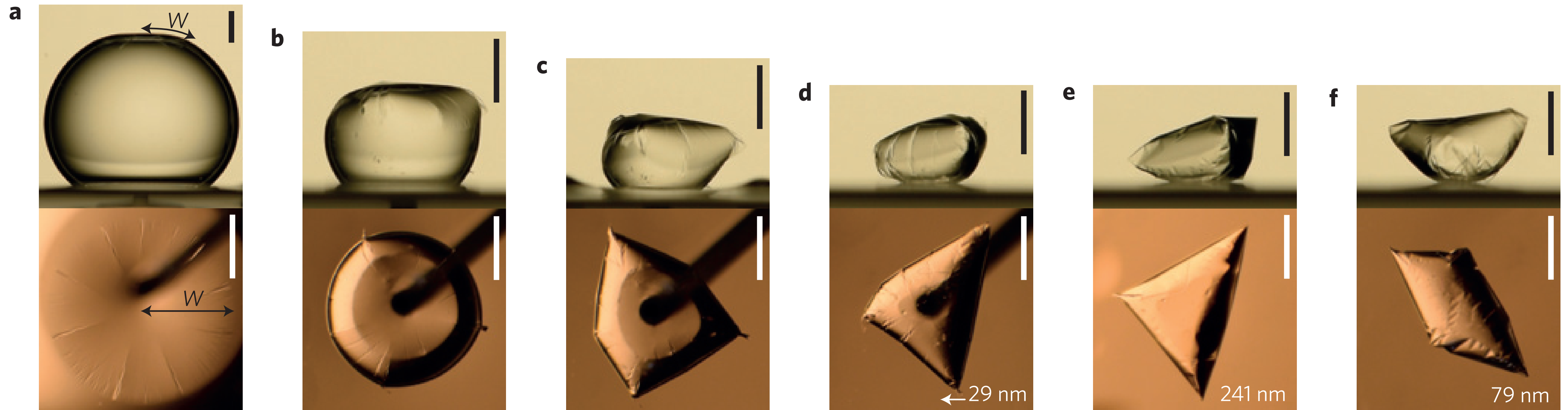
Inextensible: $\gamma < Et$

Highly bendable: $\epsilon^{-1} \gg 1$

$$\ell_m \ll t \ll \ell_m^{1/3} R^{2/3} ,$$

$$l_m = \frac{\gamma}{E}$$

Wrapping a droplet with a sheet



Thank you!