

A physics based turbulence model for shock-boundary layer interactions

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Presented by

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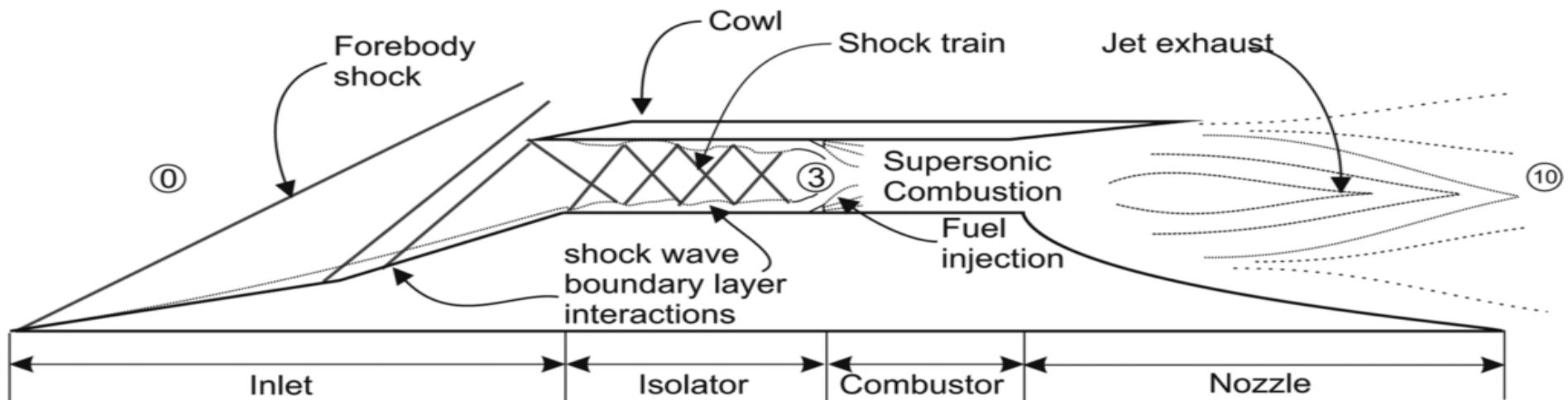


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Introduction

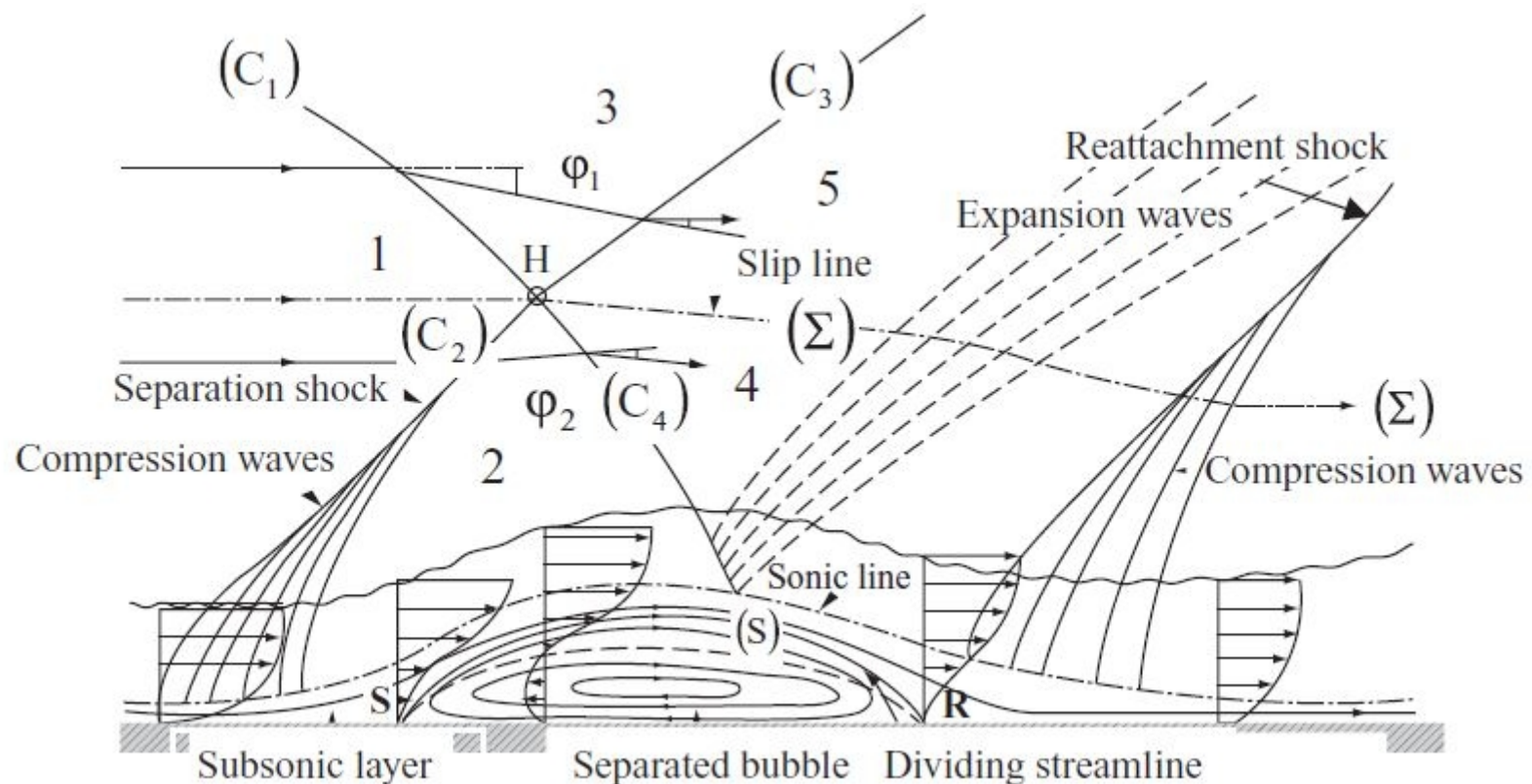
Numerical predictions of shock wave/turbulent boundary layer interactions (STBLI's) is important

- **STBLI's :**
 - High speed flows
 - e.g. intake of scramjet engines, transonic wings, fins of missiles, cone-flare configurations etc.



Schematic of Scramjet engine

- High heat loads and surface pressures
- Design of high speed vehicles
- Common studied configurations :
shock impinging on a flat plate boundary layer, compression ramps, conical flares etc.



SBLI structure on a flat plate (Ref. Babinsky and Harvey 2011)

- FANS + Eddy viscosity turbulence models**

for numerical predictions

- Favre-averaged Mass conservation $\frac{\partial \bar{\rho}}{\partial t} + \frac{\partial}{\partial x_i} (\bar{\rho} \tilde{u}_i) = 0$
- Favre-averaged Momentum $\frac{\partial}{\partial t} (\bar{\rho} \tilde{u}_i) + \frac{\partial}{\partial x_j} (\bar{\rho} \tilde{u}_j \tilde{u}_i) = -\frac{\partial P}{\partial x_i} + \frac{\partial}{\partial x_j} [\bar{t}_{ji} - \overline{\rho u_j'' u_i''}]$
- Favre-averaged Energy

$$\begin{aligned} & \frac{\partial}{\partial t} \left[\bar{\rho} \left(\tilde{e} + \frac{1}{2} \tilde{u}_i \tilde{u}_i \right) + \frac{\overline{\rho u_i'' u_i''}}{2} \right] + \frac{\partial}{\partial x_j} \left[\bar{\rho} \tilde{u}_j \left(\tilde{h} + \frac{1}{2} \tilde{u}_i \tilde{u}_i \right) + \tilde{u}_j \frac{\overline{\rho u_i'' u_i''}}{2} \right] \\ &= \frac{\partial}{\partial x_j} \left[-q_{Lj} - \overline{\rho u_j'' h''} + \bar{t}_{ji} \tilde{u}_i - \overline{\rho u_j'' \frac{1}{2} u_i'' u_i''} \right] + \frac{\partial}{\partial x_j} \left[\tilde{u}_i \left(\bar{t}_{ij} - \overline{\rho u_i'' u_j''} \right) \right] \end{aligned}$$

- * Favre-averaged Equation of state $P = \bar{\rho} R \tilde{T}$

Problems with conventional turbulence models

- Standard k-epsilon and k-omega models unable to predict
 - location and size of separation bubble
 - heat transfer rates
- Models with realizability constraints, compressibility corrections, length-scale modifications and rapid compression corrections shows little improvement
- **Main reason** : Over prediction of turbulent kinetic energy (TKE)

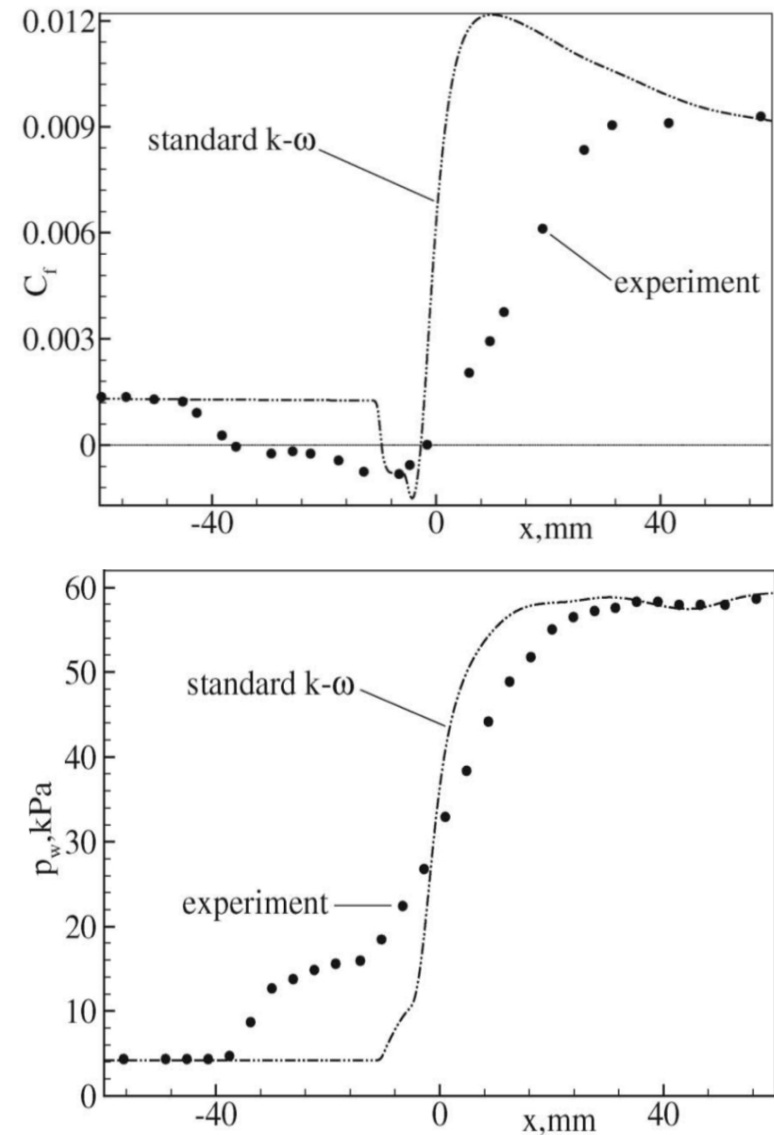


Fig. Skin friction coefficient and Wall pressure for $M = 5$ flow with Shock deflector angle of 14 deg (Ref. Pasha and Sinha 2008)

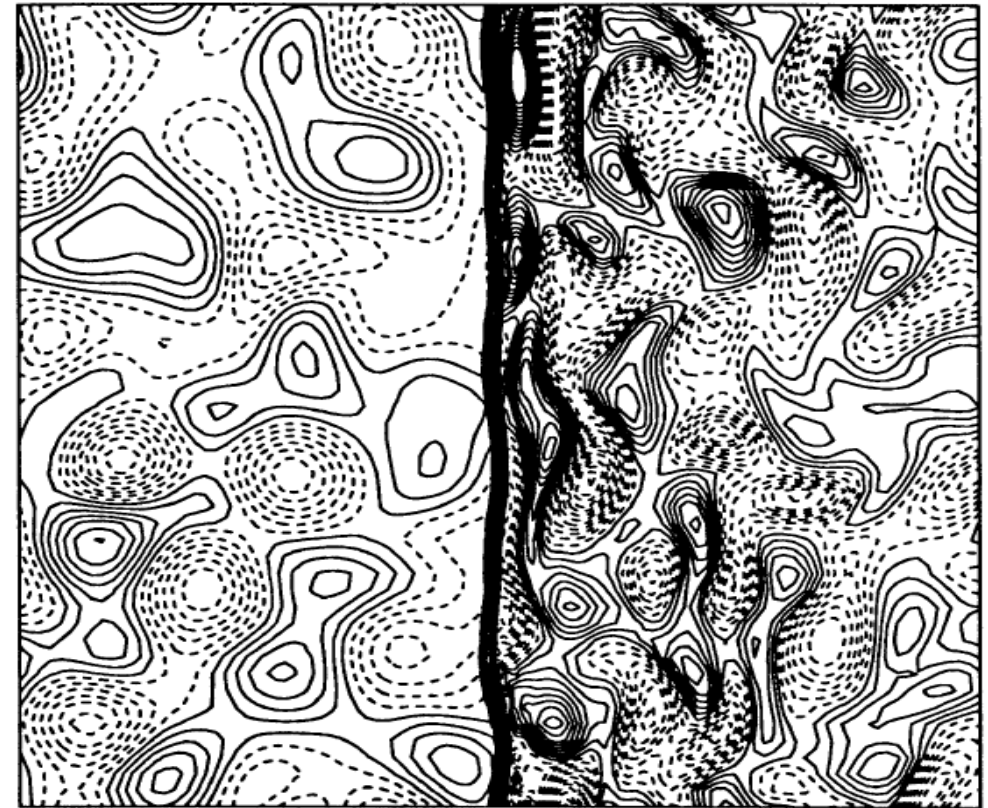
Improvements in turbulence models

- **Canonical shock turbulence interaction :**
 - Building block flow
 - Simplified mean conservation equations and turbulence model equations
 - Boundary layer velocity gradients, flow separation, streamline curvatures are neglected
 - Unsteady shock oscillations, amplification of TKE across the shock, high heat transfer rates in STI region

- **Main aim** : net effect of the strong gradients of mean quantities on turbulence
- Physics of STI
- DNS studies e.g. Mahesh et al. 1997 , Jamme et al. 2002 , Larsson et al. 2009 and 2013
- Linear interaction analysis (LIA) and Rapid distortion theory
- Reynolds stresses, TKE and vorticity amplification, variations in turbulence length scales, temperature and density variations across the shock etc.

Problem of Canonical shock/turbulence interaction

- Incoming turbulence purely vortical
- 1-D uniform steady mean flow
- Rankine-Hugoniot relations
- Turbulence amplification across the shock wave
- Unsteady shock oscillations



Flow →

Fig. STI (Lee et al. 1997)

Sinha et al. 2003 (Phys. Fluids) shock-unsteadiness k-ε model

- Turbulent kinetic energy equation :
$$\bar{\rho} \tilde{u} \frac{\partial k}{\partial x} = -\frac{2}{3} \bar{\rho} k \frac{\partial \tilde{u}}{\partial x} + \frac{2}{3} b'_1 \bar{\rho} k \frac{\partial \tilde{u}}{\partial x} - \bar{\rho} \epsilon_s$$

where $b'_1 = b_{1,\infty}(1 - e^{1-M_1})$ is the shock-unsteadiness damping parameter and $b_{1,\infty} = 0.4$

- Turbulent dissipation rate equation :
$$\bar{\rho} \tilde{u} \frac{\partial \epsilon}{\partial x} = -\frac{2}{3} c_1 \bar{\rho} \epsilon \frac{\partial \tilde{u}}{\partial x} - c_2 \frac{\bar{\rho} \epsilon^2}{k}$$

where $c_1 = 1.2 + 0.21M$ and $c_2 = 1.2$

CFD implementation : Non-local

Need to remove Non-local nature

- Upstream shock-normal Mach number along a curved shock is a non-trivial exercise
- Complicated exercise in the presence of multiple shock waves and shock-shock interactions

e.g. hypersonic cone-flare configurations and oblique shock impingements

Local formulation of TKE equation

- From conservation of mean momentum

$$\frac{\partial \tilde{u}}{\partial x} = -\frac{\tilde{u}}{\bar{\rho} \tilde{u}^2} \frac{\partial \bar{p}}{\partial x}$$

Across a normal shock :

$$\bar{p} + \bar{\rho} \tilde{u}^2 = \text{constant}$$

- assuming that $\bar{\rho} \tilde{u}^2$ scales as $\bar{p}$.

$$b_1' \frac{\partial \tilde{u}}{\partial x} = -c \frac{\tilde{u}}{\bar{p}} \frac{\partial \bar{p}}{\partial x}$$

New k-equation

$$\bar{\rho} \tilde{u} \frac{\partial k}{\partial x} = -\frac{2}{3} \bar{\rho} k \frac{\partial \tilde{u}}{\partial x} - \frac{2}{3} c \bar{\rho} k \frac{\tilde{u}}{\bar{p}} \frac{\partial \bar{p}}{\partial x} - \bar{\rho} \epsilon_s$$

Recommended value : $c = 0.194$

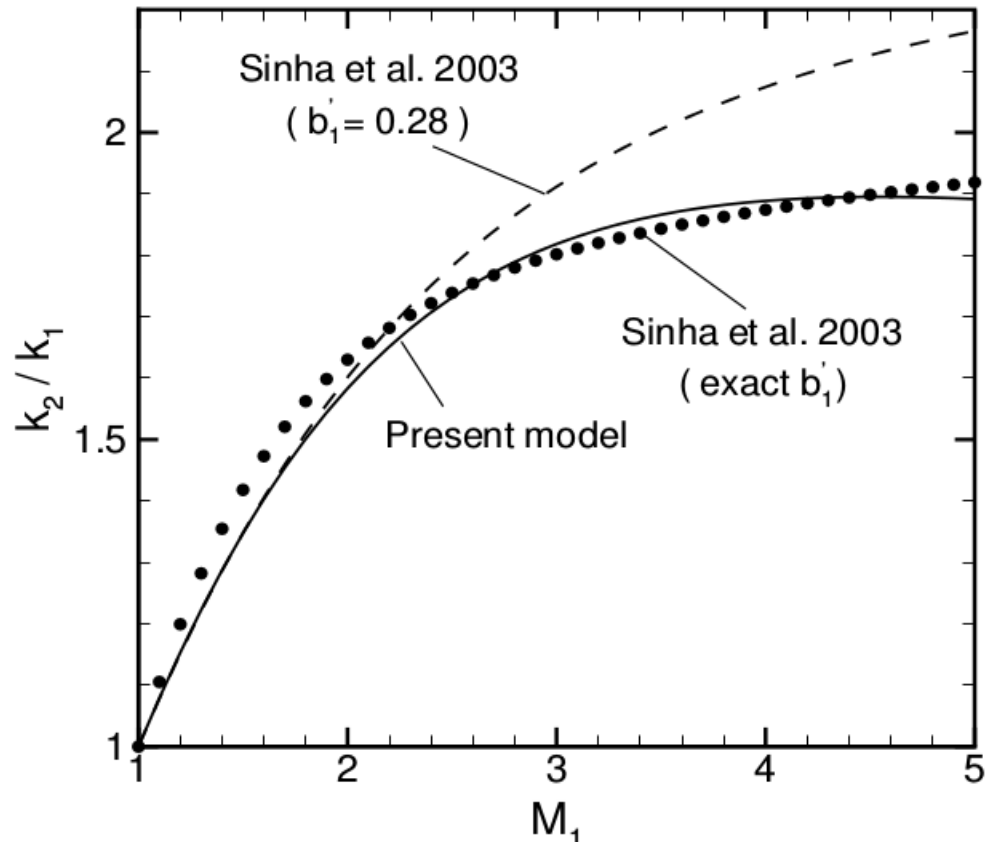


Fig. TKE amplification with upstream Mach number
(Ref. Rajee and Sinha 2016)

Extension to realistic flows

Local shock-unsteadiness TKE equation (General form)

$$\frac{\partial \rho k}{\partial t} + \frac{\partial \rho u_j k}{\partial x_j} = -\frac{2}{3} \rho k \frac{\partial u_i}{\partial x_i} - \frac{2}{3} c \rho k \frac{u_i}{p} \frac{\partial p}{\partial x_i} - \rho \epsilon$$

Model constant
 $c = 0.194$

- To be solved in the regions of shock waves in a flow-field
- To be used together with an already well-established conventional turbulence model with good performance in boundary layer flows
- eg. standard k-omega or k-epsilon models

For accurate implementation of the model :

shock detection is most crucial !!

Shock-detecting function f_s

Important requirements of the function f_s

- Must detect only shock waves and should have negligible value in other regions of the flow-field such as expansion fans
- Must be able to detect the location and extent of shock waves
eg. it must not penetrate beyond the sonic line inside the boundary layer
- Should vary smoothly from 0 (outside the shock waves) to a value of 1 (inside shock waves)

Shock-detecting function f_s of Pasha and Sinha 2008

$$f_s = \frac{1}{2} - \frac{1}{2} \tanh[12(S_{ii}\delta_0/U_\infty) + 4]$$

- Gives excellent results for incident oblique shock-boundary layer interaction cases

Standard k-epsilon model (Launder and Sharma 1974)

k-equation

$$\frac{\partial \bar{\rho} k}{\partial t} + \frac{\partial (\bar{\rho} k \tilde{u}_j)}{\partial x_j} = \tau_{ij} \frac{\partial \tilde{u}_i}{\partial x_j} + \frac{\partial}{\partial x_j} \left[\left(\mu + \frac{\mu_T}{\sigma_k} \right) \frac{\partial k}{\partial x_j} \right] - \bar{\rho} \epsilon - 2\mu \left(\frac{\partial \sqrt{k}}{\partial x_j} \right)^2$$

ε-equation

$$\begin{aligned} \frac{\partial \bar{\rho} \epsilon}{\partial t} + \frac{\partial (\bar{\rho} \epsilon \tilde{u}_j)}{\partial x_j} = & c_{\epsilon_1} \frac{\epsilon}{k} \tau_{ij} \frac{\partial \tilde{u}_i}{\partial x_j} + \frac{\partial}{\partial x_j} \left[\left(\mu + \frac{\mu_T}{\sigma_\epsilon} \right) \frac{\partial \epsilon}{\partial x_j} \right] - c_{\epsilon_2} f_2 \bar{\rho} \frac{\epsilon^2}{k} \\ & + 2 \frac{\mu \mu_T}{\rho} \left(\frac{\partial^2 \tilde{u}_i}{\partial x_j \partial x_l} \right)^2 \end{aligned}$$

Boussinesq eddy-viscosity assumption :

$$\tau_{ij} = -\bar{\rho} \widetilde{u_i'' u_j''} = 2\mu_T S_{ij} - \frac{2}{3} \mu_T \frac{\partial \tilde{u}_k}{\partial x_k} \delta_{ij} - \frac{2}{3} \rho k \delta_{ij}$$

Model constants and parameters :

$$\begin{aligned} \mu_T = c_\mu f_\mu \bar{\rho} \frac{k^2}{\epsilon} \quad & c_{\epsilon_1} = 1.44 \quad c_{\epsilon_2} = 1.92 \quad c_\mu = 0.09 \\ f_\mu = \exp \left[\frac{-3.4}{(1 + 0.02 R_t)^2} \right] \quad & f_2 = 1 - 0.3 e^{-R_t^2} \quad R_t = \frac{\bar{\rho} k^2}{\mu \epsilon} \quad \sigma_k = 1 \quad \sigma_\epsilon = 1.3 \end{aligned}$$

Blending of the two models

Production term of Standard k-epsilon model

$$P_k = \mu_T \left(2S_{ij}S_{ji} - \frac{2}{3}S_{ii}^2 \right) - \frac{2}{3}\rho k S_{ii}$$

Production term of Local SU k-epsilon model

$$P_k = -\frac{2}{3}\rho k S_{ii} - \frac{2}{3}c\rho k \frac{u_i}{p} \frac{\partial p}{\partial x_i}$$

Blending function

$$c'_k = 1 - f_s \left(1 + \frac{2c}{3} \frac{u_i}{p} \frac{\partial p}{\partial x_i} \frac{1}{c_\mu f_\mu S \zeta} \right)$$

$$\zeta = \frac{Sk}{\epsilon}$$

$$S = \left(2S_{ij}S_{ji} - \frac{2}{3}S_{ii}^2 \right)^{1/2}$$

$$f_s = \frac{1}{2} - \frac{1}{2} \tanh[12(S_{ii}\delta_0/U_\infty) + 4]$$

New k-epsilon model

Modified Production term

$$P'_k = c'_k \mu_T \left(2S_{ij}S_{ji} - \frac{2}{3}S_{ii}^2 \right) - \frac{2}{3}\rho k S_{ii}$$

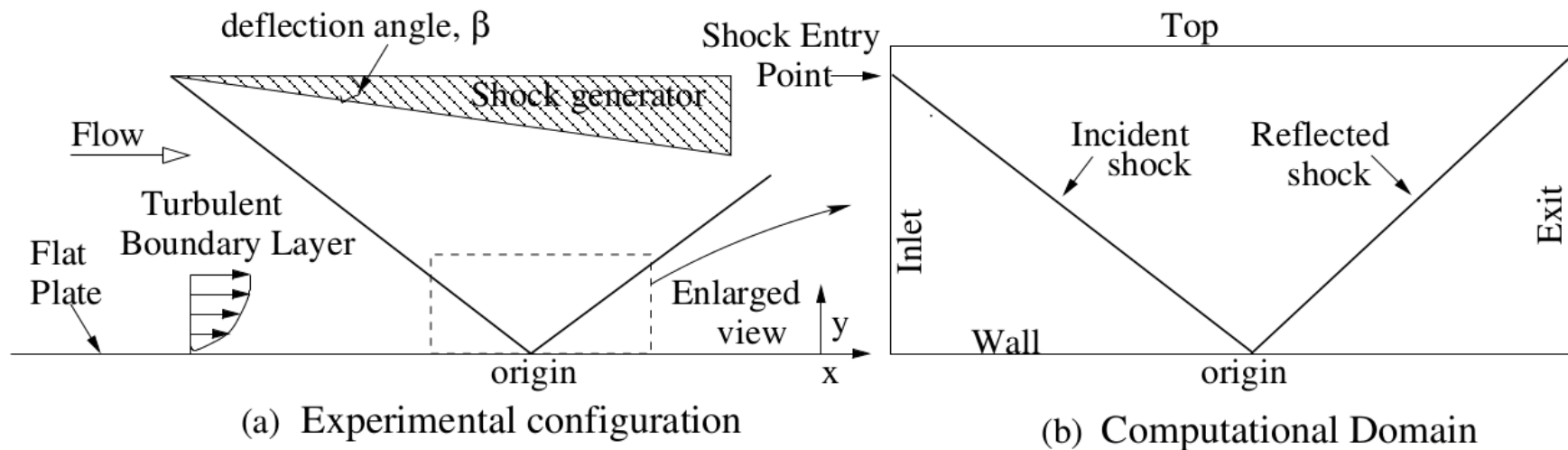
A new k-equation

$$\frac{\partial \bar{\rho} k}{\partial t} + \frac{\partial (\bar{\rho} k \tilde{u}_j)}{\partial x_j} = P'_k + \frac{\partial}{\partial x_j} \left[\left(\mu + \frac{\mu_T}{\sigma_k} \right) \frac{\partial k}{\partial x_j} \right] - \bar{\rho} \epsilon - 2\mu \left(\frac{\partial \sqrt{k}}{\partial x_j} \right)^2$$

Schulein's 2006 OSWBLI cases

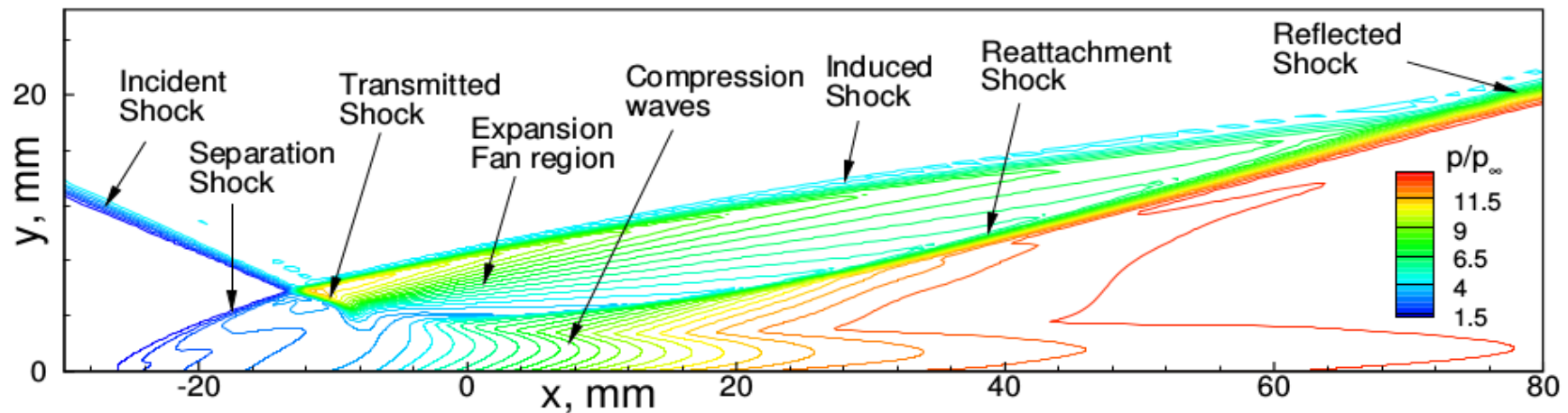
Table 2: Freestream conditions and characteristics of the incoming turbulent boundary for the incident oblique shock boundary layer interaction flows

Parameter	Shock deflection angles β , deg
	6, 10 and 14
Mach number	5
Freestream temperature, K	68.3
Freestream pressure, Pa	4008.5
Wall temperature, K	300
Reynolds number, $10^6 m^{-1}$	37
Boundary layer thickness, $10^{-3} m$	3.81
Displacement thickness, $10^{-3} m$	1.576
Momentum thickness, $10^{-4} m$	1.57

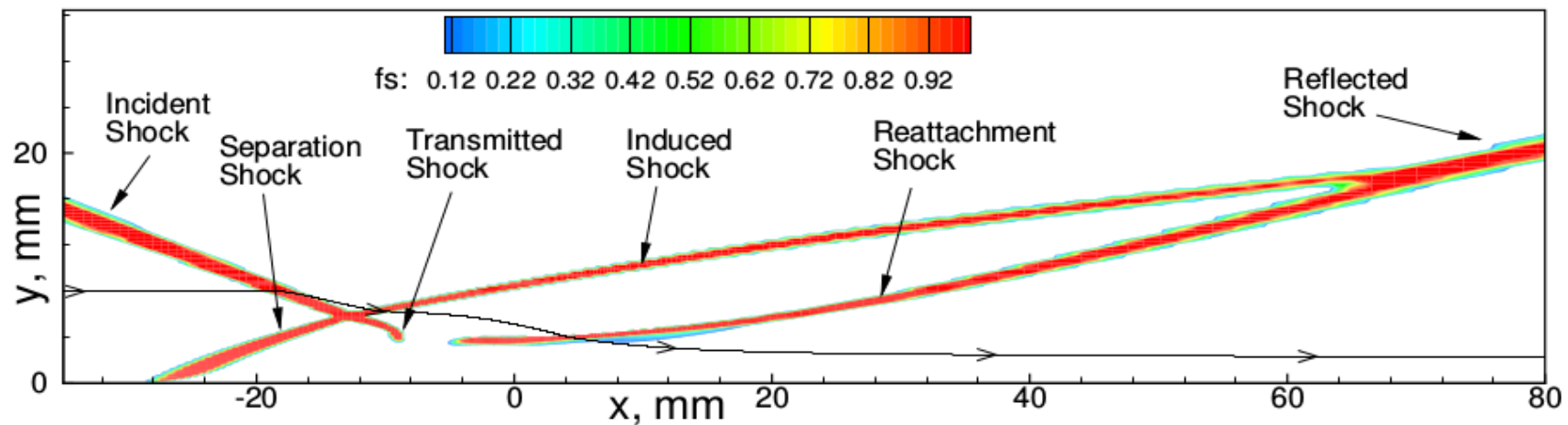


Ref. Pasha and Sinha 2008

Results with the New k-epsilon model for 14 deg Case

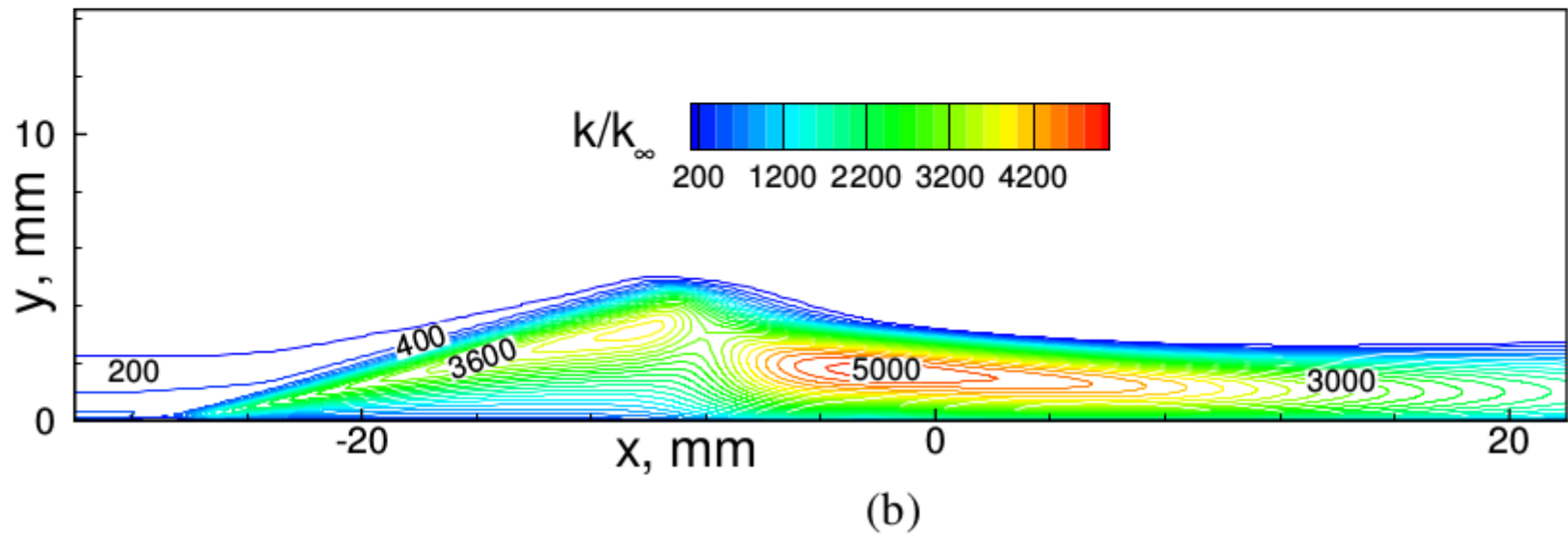
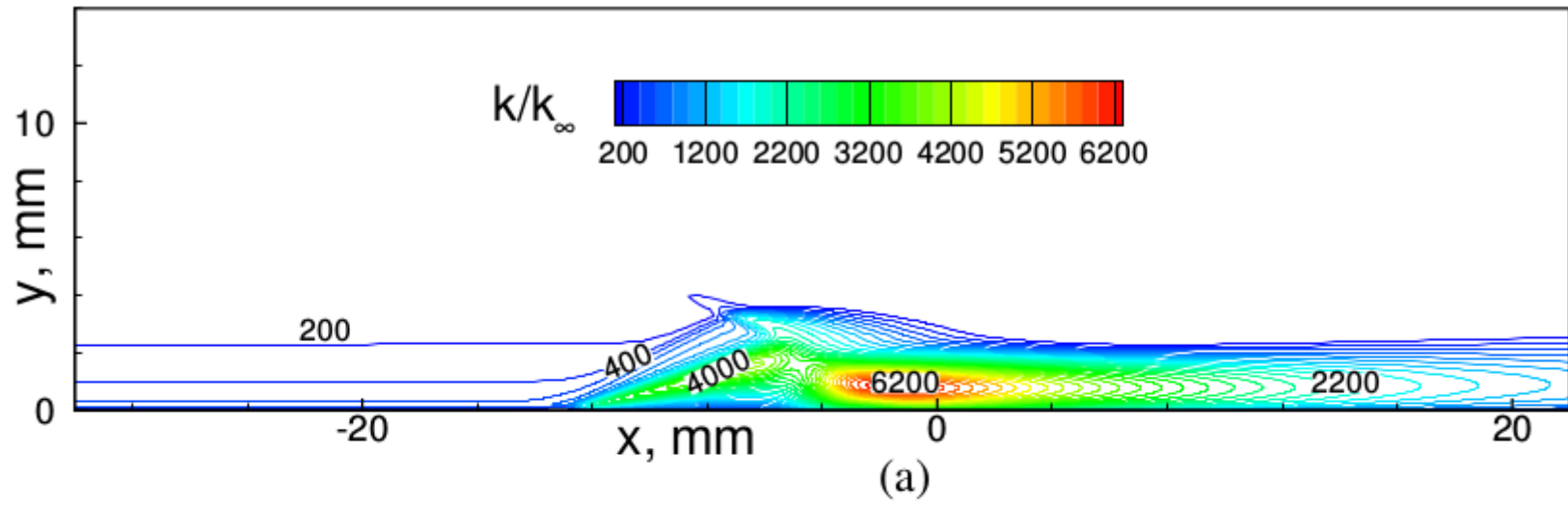


(a)



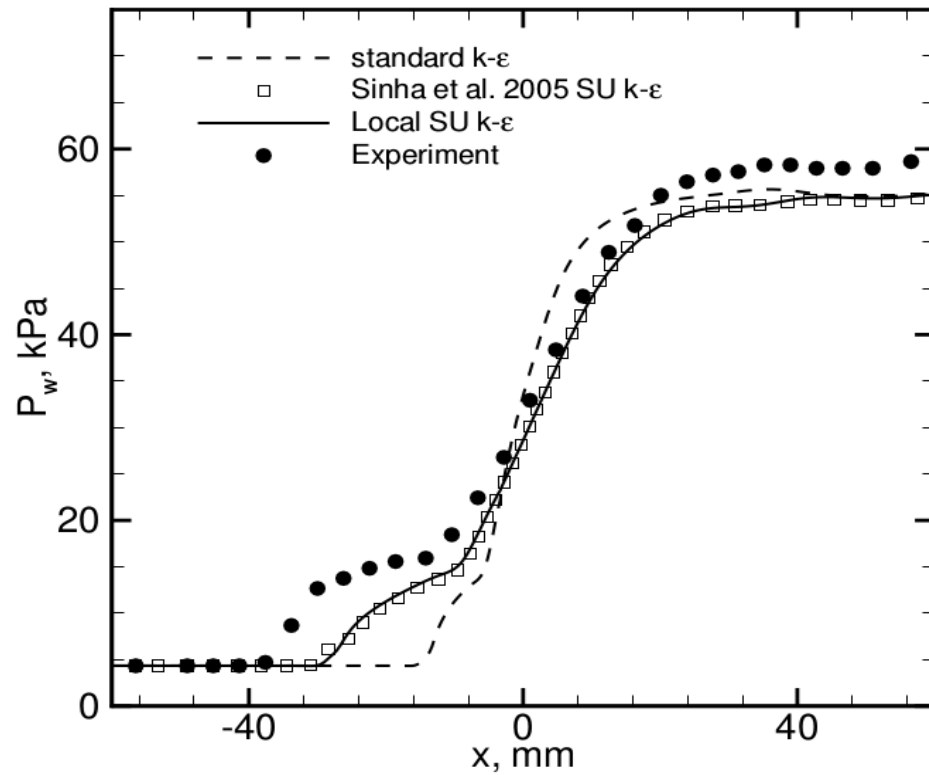
(b)

(a) Normalized Pressure and (b) fs contours in the computed flow-field

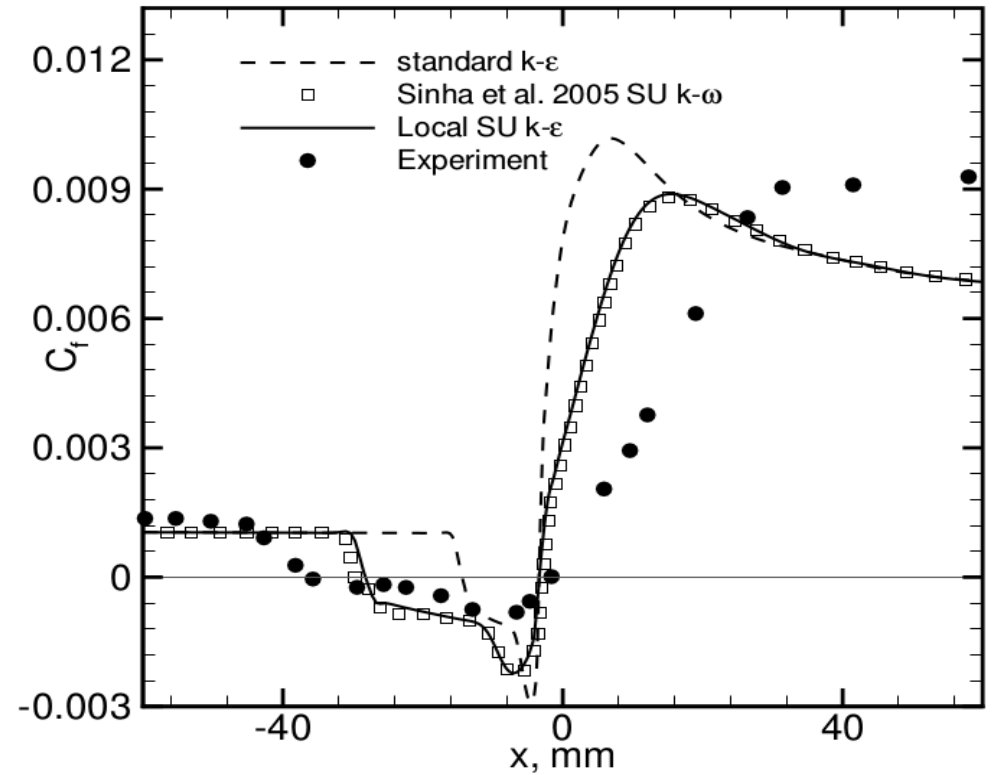


**Normalized TKE contours for the 14 deg case computed using
(a) the Standard k-epsilon model and (b) the New k-epsilon model**

Surface quantities



Wall pressure



Skin friction coefficient

Thank You