

Variable turbulent Prandtl number model for high speed flows

Subhajit Roy

Under the guidance of

Prof. Krishnendu Sinha

Department of Aerospace Engineering

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Outline

- Motivation.
- Turbulence modeling.
- Shock-unsteadiness modification.
- Variable Pr_T model.
- Results.
- Conclusion.

Motivation

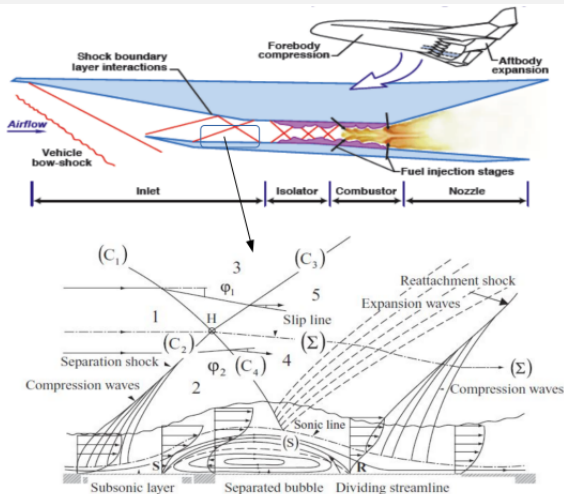


Figure : Shock/Boundary-Layer interaction (Holger Babinsky, PP-40)

Motivation

- $\frac{\partial \bar{\rho}}{\partial t} + \frac{\partial}{\partial x_i}(\bar{\rho} \tilde{u}_i) = 0.$
- $\frac{\partial}{\partial t}(\bar{\rho} \tilde{u}_i) + \frac{\partial}{\partial x_j}(\bar{\rho} \tilde{u}_j \tilde{u}_i) =$
 $-\frac{\partial \bar{P}}{\partial x_i} + \frac{\partial}{\partial x_j}(\overline{t_{ji}} - \underbrace{\overline{\rho u_j'' u_i''}}_{\text{circled}}).$
- $\frac{\partial}{\partial t}[\bar{\rho}(\tilde{e} + \frac{\tilde{u}_i \tilde{u}_i}{2})] + \frac{\partial}{\partial x_j}[\bar{\rho} \tilde{u}_j(\tilde{h} + \frac{\tilde{u}_i \tilde{u}_i}{2})] =$
 $\frac{\partial}{\partial x_j}[-q_{Lj} - \underbrace{\overline{\rho u_j'' h''}}_{\text{circled}}] + \dots.$

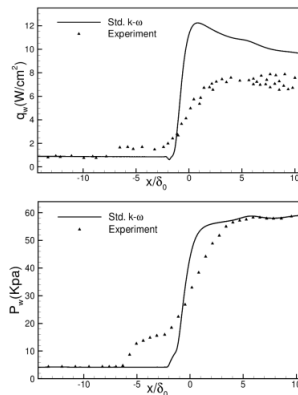


Figure : Surface properties

* Similar Results are obtained by Roy et al. 2006 ,
 Xiao et al. AIAA 2007

Turbulence modeling

- Reynolds stress tensor

$$\tau_{ij} = -\overline{\rho u_j'' u_i''} = \mu_T \left(\frac{\partial \tilde{u}_i}{\partial x_j} + \frac{\partial \tilde{u}_j}{\partial x_i} - \frac{2}{3} \frac{\partial \tilde{u}_k}{\partial x_k} \delta_{ij} \right) - \frac{2}{3} \bar{\rho} k \delta_{ij}$$

- Gradient diffusion hypothesis, $\overline{u_j'' \phi''} \propto \frac{\partial \tilde{\phi}}{\partial x_j}$

- Turbulent heat flux, $\overline{\rho u_j'' h''} = -\frac{\mu_T c_p}{Pr_T} \frac{\partial \tilde{T}}{\partial x_j}$

- Eddy viscosity, $\mu_T = \bar{\rho} \frac{k}{\omega}$

- Morkovin's hypothesis is used for Pr_T .

Morkovin's hypothesis

- Total temperature fluctuation is negligible or zero.
 - $\frac{\sqrt{\widetilde{T''^2}}}{\widetilde{T}} = -(\gamma - 1)M^2 \frac{\sqrt{\widetilde{u''^2}}}{\widetilde{u}}$
 - $R_{uT} = \overline{u''T''} / (\sqrt{\overline{u''^2}} \sqrt{\overline{T''^2}}) = -1$
 - Pr_T is taken as 0.89 by standard turbulence models.
- } Strong Reynolds Analogy

Some flaws in RANS model

- Standard model do no consider the unsteady nature of shock waves.
- Morkovin's hypothesis breakdowns behind the shock.

Shock-unsteadiness modification (Sinha et al. PoF 2003)

- TKE production term for standard $k-\omega$

$$P_k = \mu_T \left(2S_{ij}S_{ji} - \frac{2}{3}S_{ii}^2 \right) - \frac{2}{3}\bar{\rho}kS_{ii}, \text{ where}$$

$$S_{ij} = \frac{1}{2} (\partial \tilde{u}_i / \partial x_j + \partial \tilde{u}_j / \partial x_i)$$

- TKE production term for SU $k-\omega$

$$P_k = -\frac{2}{3}\rho k S_{ii} \left(1 - b'_1 \right), \text{ with}$$

$$b'_1 = \max.[0, 0.4 (1 - e^{1-M_{1n}})]$$

- $b'_1 \rightarrow 0.4$ as $M_{1n} \rightarrow \infty$ and $b'_1 = 0$ for $M_{1n} = 1$

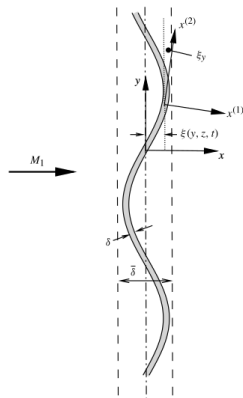


Figure : Schematic of a distorted shock (Sinha, K., JFM 2012)

Implementation of Shock-unsteadiness modification

- $c'_\mu = 1 - f_s \left[1 + \frac{b'_1}{\sqrt{3}\zeta_s} \right]$
- $\zeta_s = S/\omega$ and $S = [2S_{ij}S_{ji} - \frac{2}{3}S_{ii}^2]^{1/2}$
- $f_s = \frac{1}{2} - \frac{1}{2}\tanh [12 (S_{ii}\delta_0/U_\infty) + 4]$ (Pasha et al. IJCF 2008)

Shock-unsteadiness modification

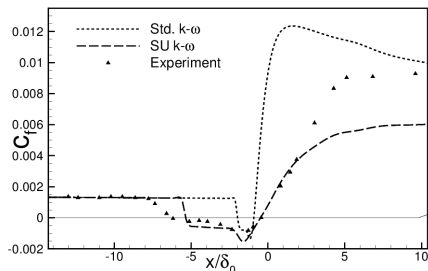


Figure : Skin friction co-efficient
(reproduced from Pasha et al. IJCF 2008)

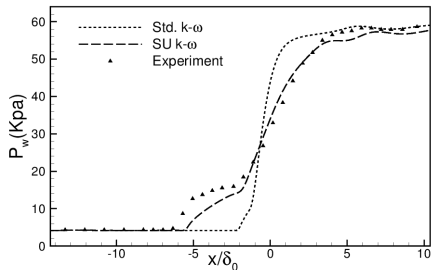


Figure : Wall pressure (reproduced from
Pasha et al. IJCF 2008)

Test cases (Schulein et al. AIAA 2006)

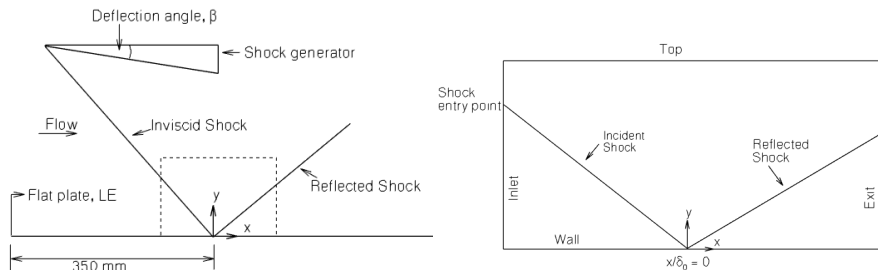


Figure : Experimental (left) and numerical configuration

Table : Test conditions

M_∞	ρ_∞	T_∞	U_∞	T_w	T_T	P_T
5	0.2043 kg/m ³	68.3 K	830 m/s	300 K	410 K	2.12 MPa

Xiao et al. model

- Two extra transport equations are solved.
- α_T is calculated from the time scale of TKE and enthalpy variance.
- $Pr_T = \frac{\nu_T}{\alpha_T}$.

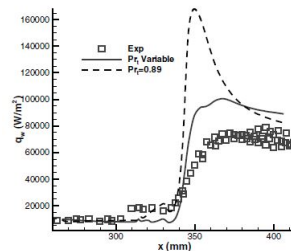


Figure : Wall heat flux (Xiao et al., AIAA 2007)

Variable Pr_T

- Shock normal Reynolds stress, $-\overline{u'^2} = \frac{4}{3}\nu_T \left(\frac{\partial \overline{u}}{\partial x} \right) - \frac{2}{3}k$
- Kinematic eddy viscosity, $\nu_T = -\frac{3}{4}\overline{u'^2} \left(\frac{\partial \overline{u}}{\partial x} \right)^{-1}$
- Eddy thermal diffusivity, $\alpha_T = -\overline{u'T'} \left(\frac{\partial \overline{T}}{\partial x} \right)^{-1}$
- Turbulent Prandtl number, $Pr_T = \frac{\nu_T}{\alpha_T} = \frac{3}{4} \frac{\overline{u'^2}}{\overline{u'T'}} \frac{\partial \overline{T}}{\partial x} \left(\frac{\partial \overline{u}}{\partial x} \right)^{-1}$

Variable Pr_T

- From conservation of total enthalpy

$$T_0 = \bar{T} + T' + \frac{(\bar{u} + u' - \xi_t)^2 + v'^2 + w'^2}{2c_p}$$

- Linearized total energy conservation across shock wave

$$T'_1 + \frac{\bar{u}_1 u'_1}{c_p} = T'_2 + \frac{\bar{u}_2 u'_2}{c_p} + \frac{\xi_t}{c_p} (\bar{u}_1 - \bar{u}_2)$$

- $T'_2 + \frac{\bar{u}_2 u'_2}{c_p} = -\frac{\xi_t}{c_p} (\bar{u}_1 - \bar{u}_2)$

- Shock normal turbulent heat flux

$$\overline{u'T'} = -\frac{\bar{u}}{c_p} \left(\overline{u'^2} + \overline{u'\xi_t}(r-1) \right)$$

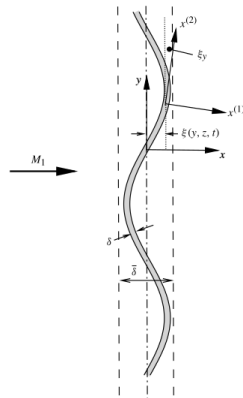


Figure : Schematic of a distorted shock
(Sinha, K., JFM 2012)

Variable Pr_T

- From Sinha et al. PoF 2003

$$\overline{u'\xi_t} = b_1 \overline{u'^2}, \text{ with } b_1 = 0.4 + 0.6e^{2(1-M_{1n})}$$

- $b_1 = 0.4 + 0.6 \left(\frac{6-r}{5} \right)^\phi$, where $\phi = 5.2$

$$\overline{u'T'} = -\frac{\bar{u}}{c_p} \overline{u'^2} (1 + b_1(r-1))$$

$$Pr_T = \frac{3}{4} \left(\frac{1}{1 + b_1(r-1)} \right)$$

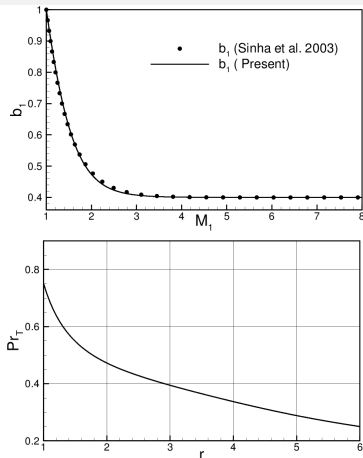


Figure : Variation of b_1 with shock normal Mach number(top) and Pr_T with density ratio across shock wave

Quadros et al. IJHFF 2016

- Turbulent heat flux saturates for high Mach numbers.
- Turbulent heat flux increases dramatically at shock wave and attains a peak value immediately behind the shock.
- Turbulent heat flux exponentially decays with distance from the shock wave.
- High heat flux values are found to persist up to $1-2 L_\epsilon$.

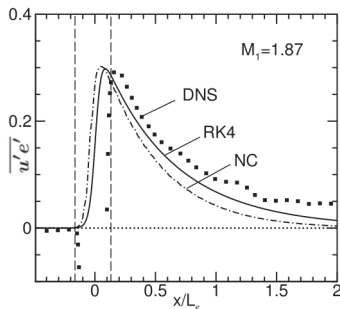


Figure : Variation of $\overline{u'e'}$ across a normal shock at $M_\infty = 1.87$ (Quadros et al. IJHFF 2016)

ψ function

- $\frac{\partial(\bar{\rho}\bar{u}\psi)}{\partial x} = \bar{\rho}\psi \frac{\partial\bar{u}}{\partial x} - \bar{\rho}(\psi - \psi_0) \frac{\bar{u}}{L_\epsilon}$
- $\frac{\psi_s}{\psi_0} = \frac{\bar{u}_2}{\bar{u}_1} = \frac{1}{r}$
- $\psi = \psi_0 + (\psi_s - \psi_0) \exp\left(-\frac{x - x_s}{L_\epsilon}\right)$

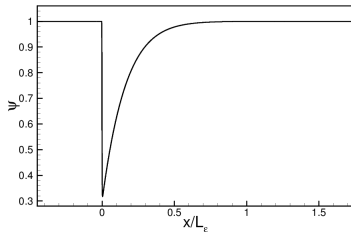


Figure : Variation of ψ across a normal shock at $M_\infty = 2.5$

ψ function

- $Pr_T = \frac{3}{4} \frac{\zeta}{1 + b_1(\frac{1}{\psi} - 1)}$
- $\zeta = 1 + \left(\frac{0.89}{0.75} - 1 \right) \exp \left(\chi \left(1 - \frac{1}{\psi} \right) \right)$
- $\frac{\partial(\bar{\rho}\psi)}{\partial t} + \frac{\partial(\bar{\rho}\tilde{u}_i\psi)}{\partial x_i} = \bar{\rho}\psi S_{ii} - \bar{\rho}(\psi - \psi_0) \frac{|\sqrt{\tilde{u}_i\tilde{u}_i}|}{L_\epsilon}$
- $L_\epsilon = \frac{\sqrt{k}}{\omega}$

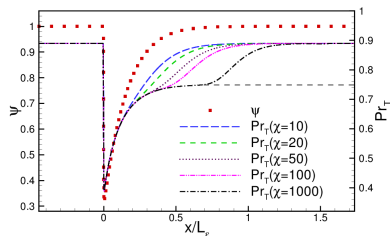


Figure : Pr_T (with different χ) variation across a normal shock at $M_\infty = 2.5$

6° Results

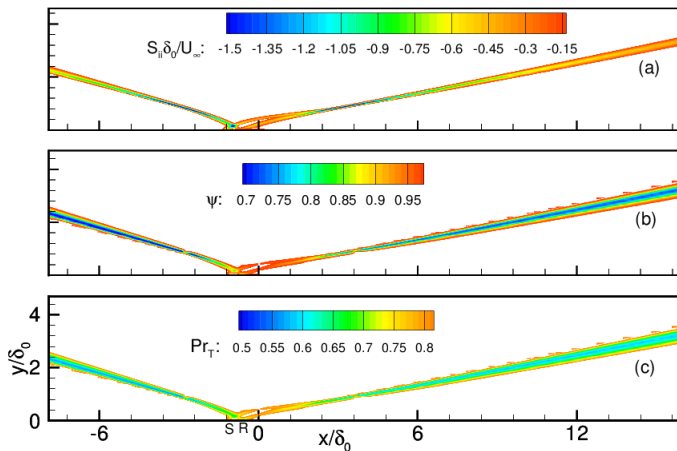
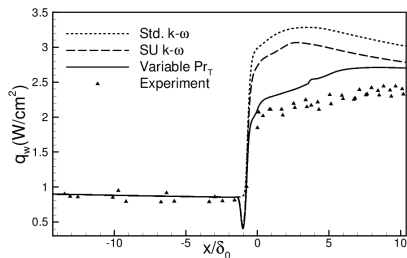
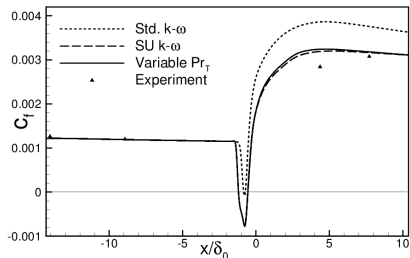


Figure : Distribution of (a) normalised mean dilatation, (b) ψ function and (c) Turbulent Prandtl number for $\beta = 6^\circ$ and $M_\infty = 5$

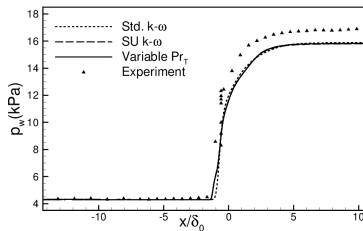
6° Results



(a) Surface heat flux



(b) Skin friction co-efficient



(c) Surface pressure

10° Results

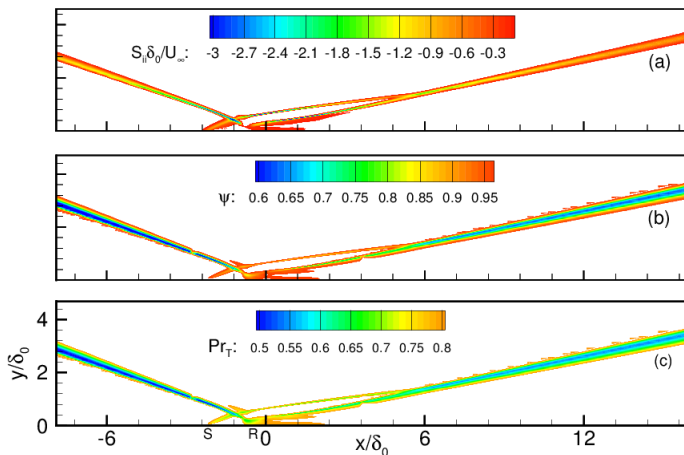
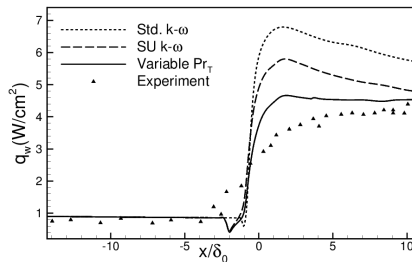
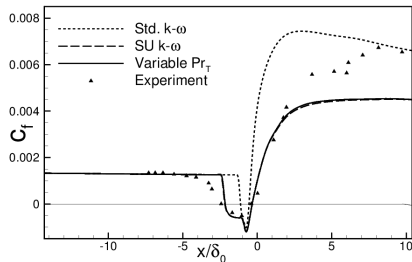


Figure : Distribution of (a) normalised mean dilatation, (b) ψ function and (c) Turbulent Prandtl number for $\beta = 10^\circ$ and $M_\infty = 5$

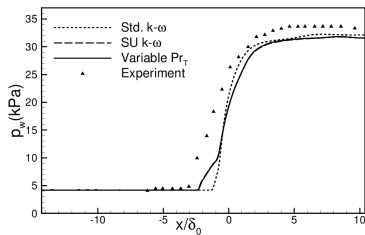
10° Results



(a) Surface heat flux



(b) Skin friction co-efficient



(c) Surface pressure

14° Results

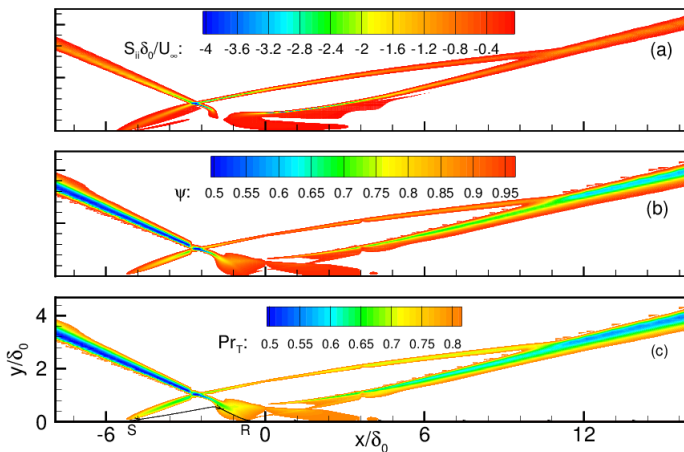
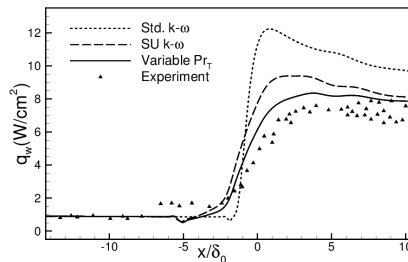
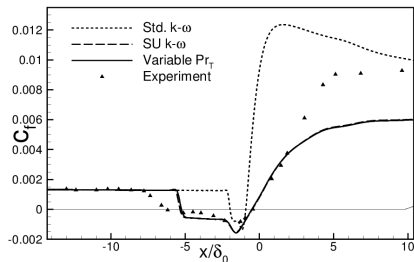


Figure : Distribution of (a) normalised mean dilatation, (b) ψ function and (c) Turbulent Prandtl number for $\beta = 14^\circ$ and $M_\infty = 5$

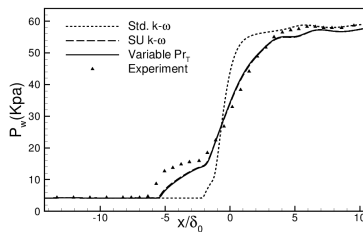
14° Results



(a) Surface heat flux



(b) Skin friction co-efficient



(c) Surface pressure

Conclusion

- Standard RANS model overestimate the peak heat flux value and underpredict the separation bubble size.
- Shock-unsteadiness modification has been found to improve the separated flow significantly.
- A new Pr_T formulation based on shock turbulence interaction physics.
- Easy implementation in CFD code and requires less computing time.

THANK YOU !