



Thermodynamic fluctuations in canonical shock-turbulence interaction

Inter-group meeting 2017

Yogesh Prasaad M S

16 January 2017

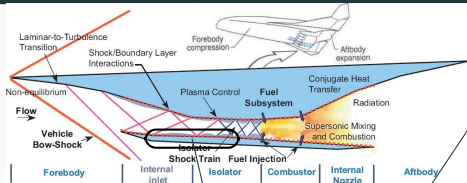
Department of Aerospace Engineering, IIT Bombay

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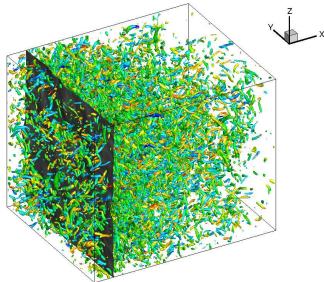
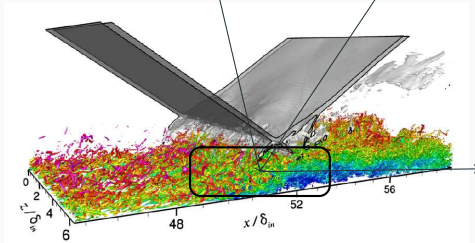
Introduction

Motivation



Stanford FPCE group

Album of fluid motion, 1982



Pirozzoli and Bernardini

3D Homogeneous/isotropic turbulence carried by 1D uniform mean flow
interacting with a normal shock wave

- Amplification of turbulent kinetic energy and enstrophy
- Substantial changes in length scales

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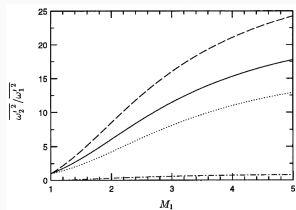
3D Homogeneous/isotropic turbulence carried by 1D uniform mean flow
interacting with a normal shock wave

- Amplification of turbulent kinetic energy and enstrophy
- Substantial changes in length scales
- Shock unsteadiness and distortion
- Amplification of thermodynamic fluctuations

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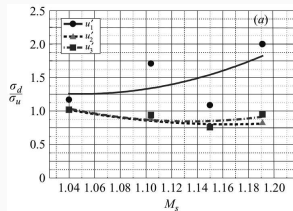
Earlier works

Theoretical - LIA



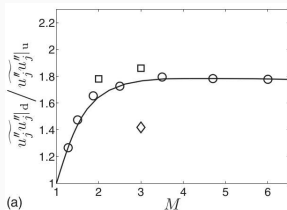
Mahesh *et al.*, TF-69, 1996

Experimental



Agui *et al.*, JFM, 2005

Numerical - DNS



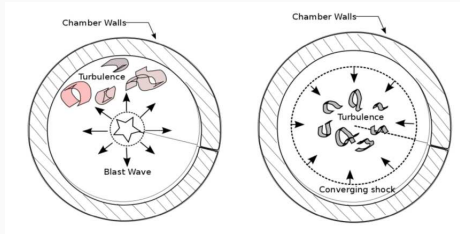
Larsson & Lele, JFM, 2009

Investigate the amplification and evolution of thermodynamic fluctuations

Objective

Investigate the amplification and evolution of thermodynamic fluctuations

$\overline{\rho'^2}$ - turbulent mixing in inertial confinement fusion

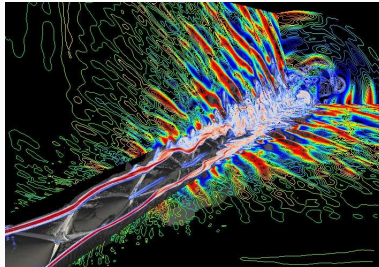


Ph.D. thesis, Ankit V. Bhagatwala, Stanford University

Objective

Investigate the amplification and evolution of thermodynamic fluctuations

$\overline{p'^2}$ - noise generation in supersonic overexpanded jets

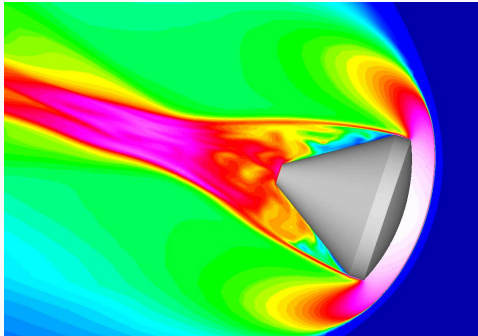


Ph.D. thesis, Nicolas de Cacqueray, LMFA, University of Lyon
Snapshot of density gradient magnitude in gray scale, azimuthal vorticity in color scale in the jet, and fluctuating pressure in color scale outside the jet. Pressure levels from -8000 to 8000 Pa. $Re = 10^5$, $M_e = 3.30$, $p = 0.5 \times 10^5$ Pa and $T = 360$ K.

Objective

Investigate the amplification and evolution of thermodynamic fluctuations

$\overline{T'^2}$ - surface heat flux in reentry vehicles, combustion, etc.



Prof. Krishnendu Sinha, Dept. of Aerospace Engg. IIT Bombay

Image from: <http://www.aem.umn.edu/people/others/krish/>

Linear interaction analysis

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VOLUME 20

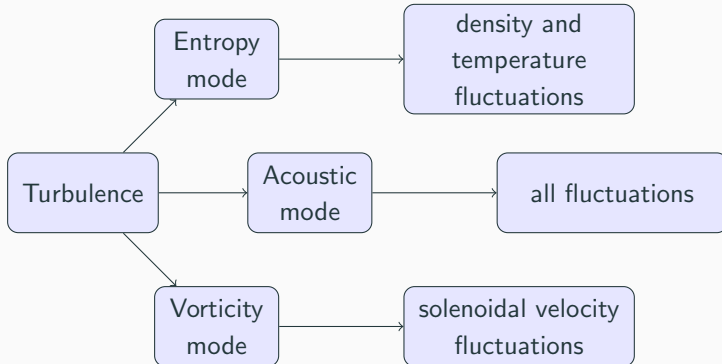
OCTOBER, 1953

NUMBER 10

Turbulence in Supersonic Flow*

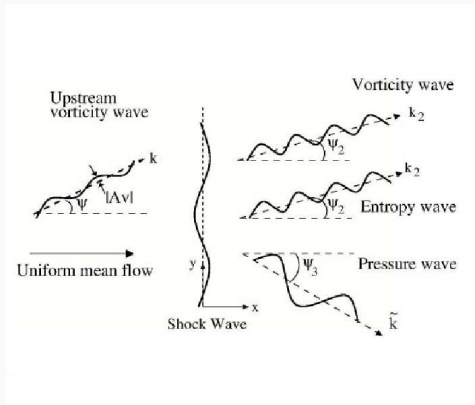
LESLIE S. G. KOVÁSZNAY†

The Johns Hopkins University



Linear Interaction Analysis (Ribner, 1954)

- Linearized Euler equations
- Linearized R-H boundary conditions
- Shock - thin unsteady discontinuity
- Downstream solution - linear superposition of modes



Quadros *et al.* FTC 2016

Linear Interaction Analysis (Ribner, 1954)

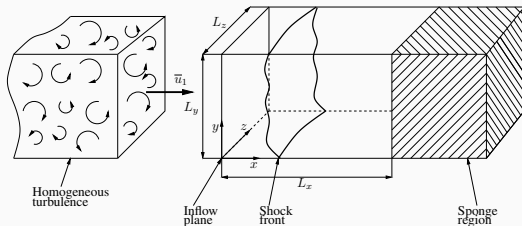
$$\frac{1}{A_v} \frac{\rho_2'}{\bar{\rho}_2} = \underbrace{\frac{K_c}{\gamma} e^{i\tilde{k}x} e^{ik(l y - m\bar{u}_1 t)}}_{\text{Acoustic}} + \underbrace{Q_c e^{ik(mrx + ly - m\bar{u}_1 t)}}_{\text{Entropy}}$$

$$\left(\frac{\overline{\rho_2'^2}}{\bar{\rho}^2} \right)_{2D} = \left[\underbrace{\frac{|K_c|^2}{\gamma^2} e^{i(\tilde{k} - \tilde{k}^*)x}}_{\text{Acoustic-Acoustic}} + \underbrace{|Q_c|^2}_{\text{Entropy-Entropy}} + \underbrace{\frac{K_c^*}{\gamma} Q_c e^{-i(\tilde{k}^* - \bar{k})x} + \frac{K_c}{\gamma} Q_c^* e^{i(\tilde{k} - \bar{k})x}}_{\text{Acoustic-Entropy}} \right] |A_v|^2$$

$$(\overline{f'g'})_{3D} = 4\pi \int_{k=0}^{\infty} \int_{\psi=0}^{\pi/2} (\overline{f'g'})_{2D} k^2 \sin(\psi) d\psi dk$$

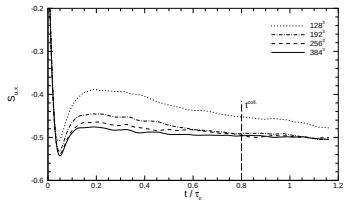
Direct Numerical Simulation

Direct numerical simulation

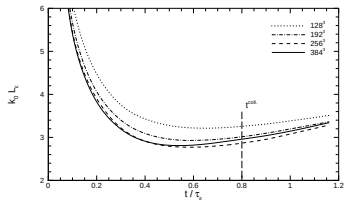


- C++ code “hybrid” developed by Prof. Johan Larsson, UMD
- Shock-captured but turbulence is fully resolved
- Ideal gas law and power-law for viscosity ($\mu \sim T^{0.75}$)
- Inflow: Temporally decaying turbulence
- Outflow: Non-reflecting + sponge
- Periodic conditions in transverse and spanwise directions

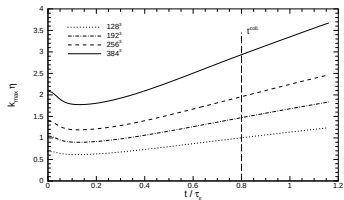
Homogeneous/isotropic turbulence



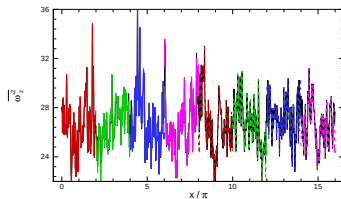
Skewness of velocity derivatives



Dissipation length scale



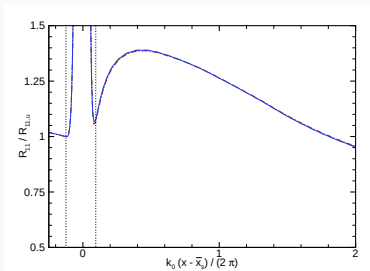
Kolmogorov length scale



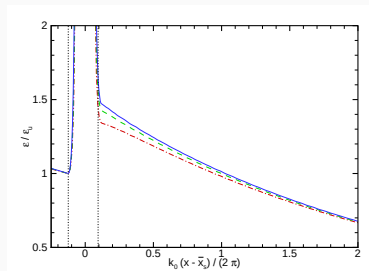
Blended database
transverse vorticity variance

Cases and validation

M_1	M_t	Re_λ	Re_{L_ε}	L_ε/η	$\Delta x_{2,3}/\Delta x_{1,s}$	δ/η	Grid
1.23	0.15	33	133	45	1.33	0.90	882×384^2
1.50	0.15	33	134	45	2.0	0.41	1042×384^2
2.50	0.15	33	135	46	2.5	0.14	1142×384^2
3.50	0.15	33	136	46	3.0	0.08	1313×384^2



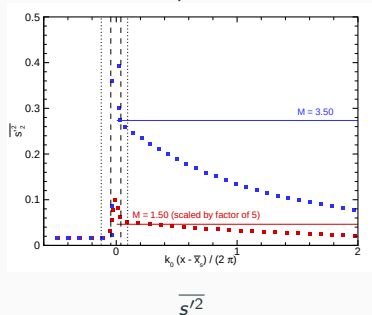
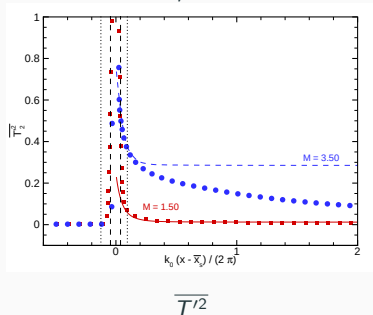
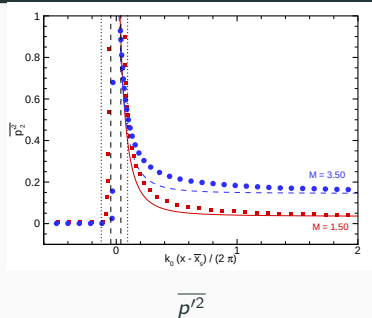
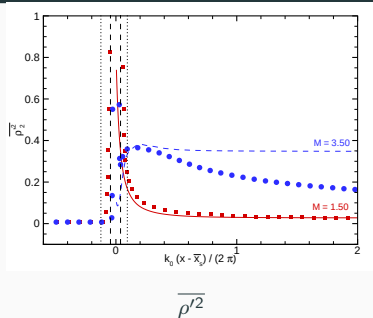
R_{11}



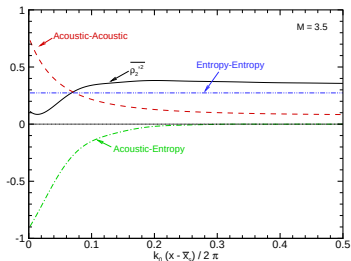
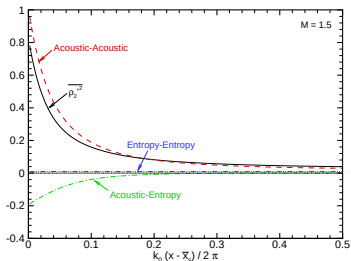
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Thermodynamic analysis

Thermodynamic variance against space

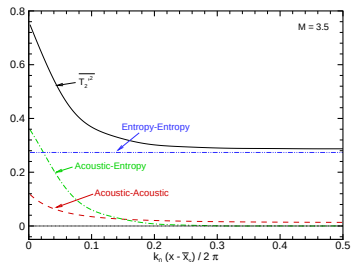
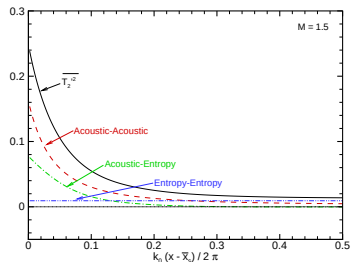


Modal decomposition



$\overline{\rho'^2} - M1.50$

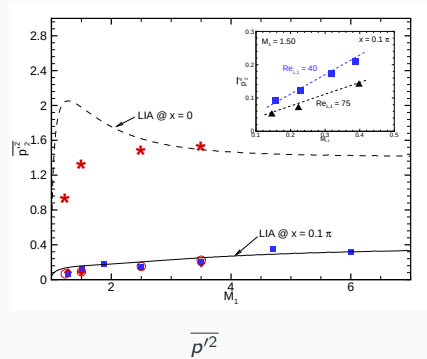
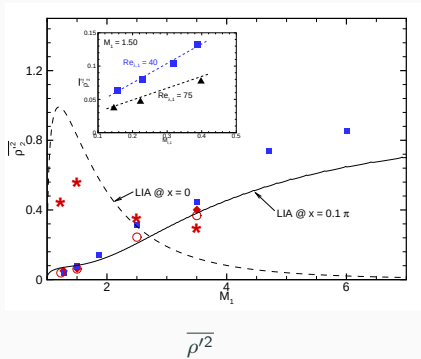
$\overline{\rho'^2} - M3.50$



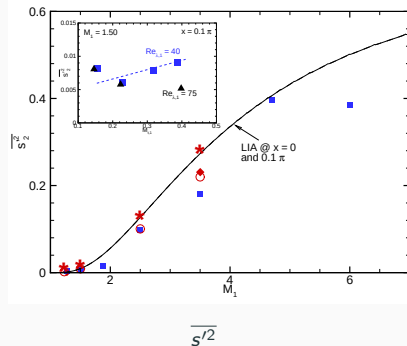
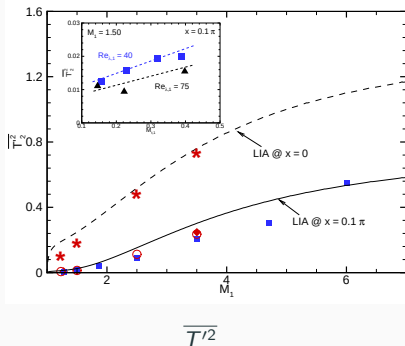
$\overline{T'^2} - M1.50$

$\overline{T'^2} - M3.50$

Variation against Mach number



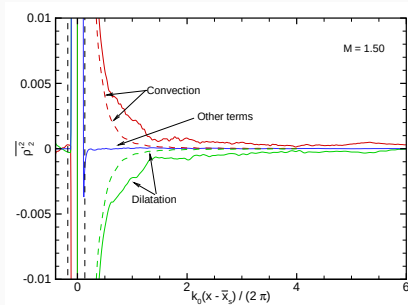
Variation against Mach number



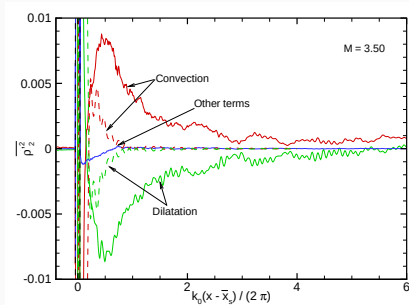
Budget analysis

Budgets

$$\underbrace{\tilde{u}_1 \partial_1 (\overline{\rho'^2})}_{\text{Convection}} = \underbrace{-2\overline{\rho \rho'} \partial_k u_k''}_{\text{Dilatation}} + \text{O.T.}$$



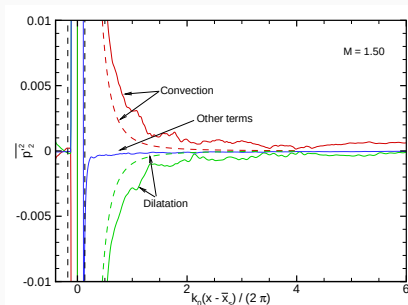
$M = 1.5$



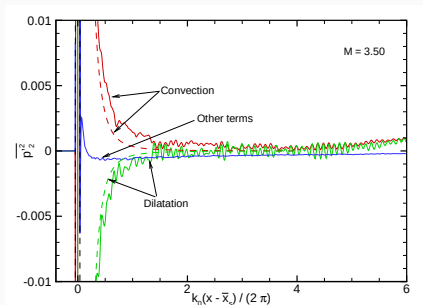
$M = 3.5$

Budgets

$$\underbrace{\tilde{u}_1 \partial_1 (\overline{p'^2})}_{\text{Convection}} = \underbrace{-2\gamma \overline{p p'} \partial_k u_k''}_{\text{Dilatation}} - \underbrace{2(\gamma - 1) \overline{\partial_k p' \kappa \partial_k T''}}_{\text{Dissipation}} + \text{O.T.}$$



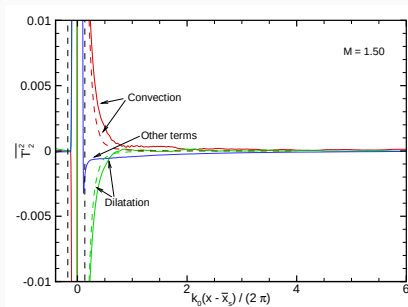
$M = 1.5$



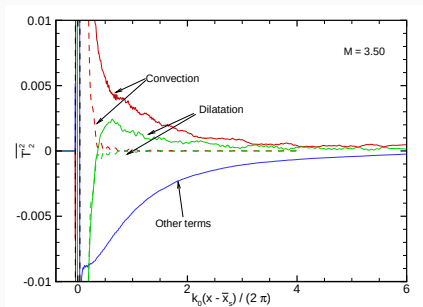
$M = 3.5$

Budgets

$$\underbrace{\partial_k(\tilde{u}_k \overline{\rho T''^2})}_{\text{Convection}} = \underbrace{-(2/c_v) \overline{p T'' \partial_k u_k''}}_{\text{Dilatation}} - \underbrace{(2/c_v) \overline{\partial_k T'' \kappa \partial_k T''}}_{\text{Dissipation}} + \text{O.T.}$$



$M = 1.5$



$M = 3.5$

Conclusion

Thermodynamic analysis

Variance	Near-Field	Far-Field
Density	moderate	low
Pressure	moderate	all
Temperature	all	low
Entropy	all	very low

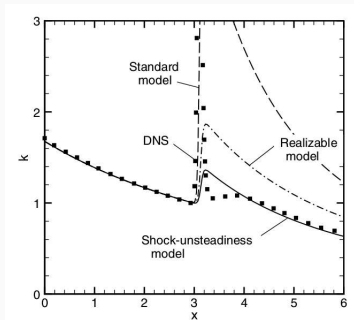
Acoustic mode is predicted well throughout

Vorticity and entropy modes are predicted well just behind the shock

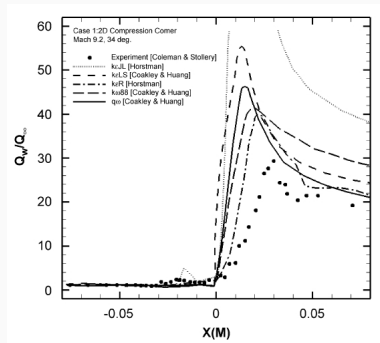
Budget analysis

- Correlation with dilatation - dominant term
- Comparable effect of dissipation - temperature and entropy variance
- LIA predicts acoustic mode - very well
- Can model dilatation correlation using LIA
- Can model dissipation effects similar to the work of Sinha JFM 2012.

Applications



Sinha and Balasridhar, AIAA J., 2013
 $M = 1.5$, $M_t = 0.17$, $Re_\lambda = 6.7$



Roy and Blottner, *Prog. Aero. Sci.*,
 2006
 $M = 9.2$, 34° compression corner

Thank you